Chapter 2

Motion in One Dimension

2.1 The Important Stuff

2.1.1 Position, Time and Displacement

We begin our study of motion by considering objects which are very small in comparison to the size of their movement through space. When we can deal with an object in this way we refer to it as a **particle**. In this chapter we deal with the case where a particle moves along a straight line.

The particle's location is specified by its **coordinate**, which will be denoted by x or y. As the particle moves, its coordinate changes with the time, t. The change in position from x_1 to x_2 of the particle is the **displacement** Δx, with $\Delta x = x_2 - x_1$.

2.1.2 Average Velocity and Average Speed

When a particle has a displacement Δx in a change of time Δt, its **average velocity** *for that time interval* is

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{2.1}$$

The **average speed** of the particle is absolute value of the average velocity and is given by

$$\overline{s} = \frac{\text{Distance travelled}}{\Delta t} \tag{2.2}$$

In general, the value of the average velocity for a moving particle depends on the initial and final times for which we have found the displacements.

2.1.3 Instantaneous Velocity and Speed

We can answer the question "how fast is a particle moving at a particular time t?" by finding the **instantaneous velocity**. This is the limiting case of the average velocity when the time interval Δt include the time t and is as small as we can imagine:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$

The **instantaneous speed** is the absolute value (magnitude) of the instantaneous velocity.

If we make a plot of x vs. t for a moving particle the instantaneous velocity is the slope of the tangent to the curve at any point.

2.1.4 Acceleration

When a particle's velocity changes, then we way that the particle undergoes an **acceleration**.

If a particle's velocity changes from v_1 to v_2 during the time interval t_1 to t_2 then we define the **average acceleration** as

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (2.4)$$

As with velocity it is usually more important to think about the **instantaneous acceleration**, given by

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (2.5)$$

If the acceleration a is positive it means that the velocity is instantaneously *increasing*; if a is negative, then v is instantaneously *decreasing*. Oftentimes we will encounter the word **deceleration** in a problem. This word is used when the sense of the acceleration is opposite that of the instantaneous velocity (the motion). Then the *magnitude* of acceleration is given, with its direction being understood.

2.1.5 Constant Acceleration

A very useful *special case* of accelerated motion is the one where the acceleration a is constant. For this case, one can show that the following are true:

$$v = v_0 + at \qquad (2.6)$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad (2.7)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2.8)$$

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad (2.9)$$

In these equations, we mean that the particle has position x_0 and velocity v_0 at time $t = 0$; it has position x and velocity v at time t.

These equations are valid *only* for the case of constant acceleration.

2.1.6 Free Fall

An object tossed up or down near the surface of the earth has a constant downward acceleration of magnitude $9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$. This number is always denoted by g. Be very careful about the sign; in a coordinate system where the y axis points straight up, the acceleration of a freely–falling object is

$$a_y = -9.80\,\frac{\mathrm{m}}{\mathrm{s}^2} = -g \tag{2.10}$$

Here we are assuming that the air has no effect on the motion of the falling object. For an object which falls for a long distance this can be a bad assumption.

Remember that an object in free–fall has an acceleration equal to $-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$ while it is moving up, while it is moving down, while it is at maximum height... always!

2.2 Worked Examples

2.2.1 Average Velocity and Average Speed

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1. Boston Red Sox pitcher Roger Clemens could routinely throw a fastball at a horizontal speed of $160\,\frac{\mathrm{km}}{\mathrm{hr}}$. How long did the ball take to reach home plate $18.4\,\mathrm{m}$ away? [HRW5 2-4]

We assume that the ball moves in a horizontal straight line with an average speed of 160 km/hr. Of course, in reality this is not quite true for a thrown baseball.

We are given the average velocity of the ball's motion and also a particular displacement, namely $\Delta x = 18.4\,\mathrm{m}$. Equation 2.1 gives us:

$$\overline{v} = \frac{\Delta x}{\Delta t} \qquad \Longrightarrow \qquad \Delta t = \frac{\Delta x}{\overline{v}}$$

But before using it, it might be convenient to change the units of $\overline{v}$. We have:

$$\overline{v} = 160\,\frac{\mathrm{km}}{\mathrm{hr}} \cdot \left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right) \cdot \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right) = 44.4\,\frac{\mathrm{m}}{\mathrm{s}}$$

Then we find:

$$\Delta t = \frac{\Delta x}{\overline{v}} = \frac{18.4\,\mathrm{m}}{44.4\,\frac{\mathrm{m}}{\mathrm{s}}} = 0.414\,\mathrm{s}$$

The ball takes 0.414 seconds to reach home plate.

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2. Taking the Earth's orbit to be a circle of radius $1.5 \times 10^8\,\mathrm{km}$, determine the speed of the Earth's orbital motion in (a) meters per second and (b) miles per second. [Wolf 2-18]

(a) This is not straight line motion of course, but we can sill find an average speed by dividing the distance traveled (around a circular path) by the time interval. Here, the distance traveled by the Earth as it goes once around the Sun is the circumference of the orbit,

$$C = 2\pi R = 2\pi(1.5 \times 10^8\,\mathrm{km}) = 9.42 \times 10^8\,\mathrm{km} = 9.42 \times 10^{11}\,\mathrm{m}$$

and the time interval over which that takes place is one year,

$$1\,\mathrm{yr} = 365.25\,\mathrm{day}\left(\frac{24\,\mathrm{hr}}{1\,\mathrm{day}}\right)\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{hr}}\right) = 3.16 \times 10^7\,\mathrm{s}$$

so the average speed is

$$s = \frac{C}{t} = \frac{9.42 \times 10^{11}\,\mathrm{m}}{3.16 \times 10^7\,\mathrm{s}} = 2.99 \times 10^4\,\frac{\mathrm{m}}{\mathrm{s}}$$

(b) To convert this to $\frac{\mathrm{mi}}{\mathrm{s}}$, use $1\,\mathrm{mi} = 1.609\,\mathrm{km}$. Then

$$s = \left(2.99 \times 10^4\,\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{1\,\mathrm{mi}}{1.609 \times 10^3\,\mathrm{m}}\right) = 18.6\,\frac{\mathrm{mi}}{\mathrm{s}}$$

2.2.2 Acceleration

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3. An electron moving along the x axis has a position given by $x = (16te^{-t})\,\mathrm{m}$, where t is in seconds. How far is the electron from the origin when it momentarily stops? [HRW6 2-20]

To find the velocity of the electron as a function of time, take the first derivative of $x(t)$:

$$v = \frac{dx}{dt} = 16e^{-t} - 16te^{-t} = 16e^{-t}(1-t)\,\frac{\mathrm{m}}{\mathrm{s}}$$

again where t is in seconds, so that the units for v are $\frac{\mathrm{m}}{\mathrm{s}}$.

Now the electron "momentarily stops" when the velocity v is zero. From our expression for v we see that this occurs at $t = 1\,\mathrm{s}$. At this particular time we can find the value of x:

$$x(1\,\mathrm{s}) = 16(1)e^{-1}\,\mathrm{m} = 5.89\,\mathrm{m}$$

The electron was 5.89 m from the origin when the velocity was zero.

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4. (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when if ever is the particle's velocity zero? (b) When is its acceleration a zero? (c) When is a negative? Positive? (d) Graph $x(t)$, $v(t)$, and $a(t)$. [HRW5 2-28]

(a) From Eq. 2.3 we find $v(t)$ from $x(t)$:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(20t - 5t^3) = 20 - 15t^2$$

where, if t is in seconds then v will be in $\frac{\mathrm{m}}{\mathrm{s}}$. The velocity v will be zero when

$$20 - 15t^2 = 0$$

which we can solve for t:

$$15t^2 = 20 \qquad \Longrightarrow \qquad t^2 = \frac{20}{15} = 1.33\,\mathrm{s}^2$$

(The units s^2 were inserted since we know t^2 must have these units.) This gives:

$$t = \pm 1.15\,\mathrm{s}$$

(We should be careful... t may be meaningful for negative values!)

(b) From Eq. 2.5 we find $a(t)$ from $v(t)$:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(20 - 15t^2) = -30t$$

where we mean that if t is given in seconds, a is given in $\frac{\mathrm{m}}{\mathrm{s}^2}$. From this, we see that a can be zero only at $t = 0$.

(c) From the result is part (b) we can also see that a is negative whenever t is positive. a is positive whenever t is negative (again, assuming that $t < 0$ has meaning for the motion of this particle).

(d) Plots of $x(t)$, $v(t)$, and $a(t)$ are given in Fig. 2.1.

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5. In an arcade video game a spot is programmed to move across the screen according to $x = 9.00t - 0.750t^3$, where x is distance in centimeters measured from the left edge of the screen and t is time in seconds. When the spot reaches a screen edge, at either $x = 0$ or $x = 15.0\,\mathrm{cm}$, t is reset to 0 and the spot starts moving again according to $x(t)$. (a) At what time after starting is the spot instantaneously at rest? (b) Where does this occur? (c) What is its acceleration when this occurs? (d) In what direction is it moving just prior to coming to rest? (e) Just after? (f) When does it first reach an edge of the screen after $t = 0$? [HRW5 2-31]

(a) This is a question about the instantaneous velocity of the spot. To find $v(t)$ we calculate:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(9.00t - 0.750t^3) = 9.00 - 2.25t^2$$

where this expression will give the value of v in $\frac{\mathrm{cm}}{\mathrm{s}}$ when t is given in seconds.

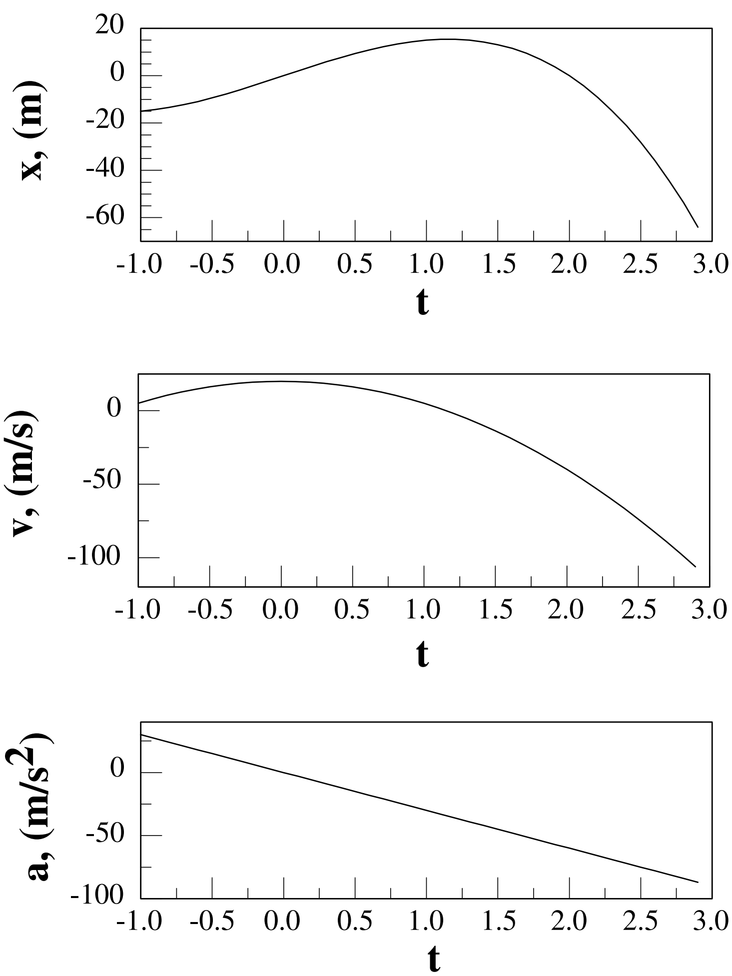

Figure 2.1: Plot of $x(t)$, $v(t)$, and $a(t)$ for Example 4.

We want to know the value of t for which v is zero, i.e. the spot is instantaneously at rest. We solve:

$$9.00 - 2.25t^2 = 0 \qquad \Longrightarrow \qquad t^2 = \frac{9.00}{2.25} = 4.00\,\mathrm{s}^2$$

(Here we have filled in the proper units for t^2 since by laziness they were omitted from the first equations!) The solutions to this equation are

$$t = \pm 2.00\,\mathrm{s}$$

but since we are only interested in times after the clock starts at $t = 0$, we choose $t = 2.00\,\mathrm{s}$.

(b) In this part we are to find the value of x at which the instantaneous velocity is zero. In part (a) we found that this occurred at $t = 3.00\,\mathrm{s}$ so we calculate the value of x at $t = 2.00\,\mathrm{s}$:

$$x(2.00\,\mathrm{s}) = 9.00 \cdot (2.00) - 0.750 \cdot (2.00)^3 = 12.0\,\mathrm{cm}$$

(where we have filled in the units for x since *centimeters* are implied by the equation). The dot is located at $x = 12.0\,\mathrm{cm}$ at this time. (And recall that the width of the screen is 15.0 cm.)

(c) To find the (instantaneous) acceleration at all times, we calculate:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(9.00 - 2.25t^2) = -4.50t$$

where we mean that if t is given in seconds, a will be given in $\frac{\mathrm{m}}{\mathrm{s}^2}$. At the time in question ($t = 2.00\,\mathrm{s}$) the acceleration is

$$a(t = 2.00\,\mathrm{s}) = -4.50 \cdot (2.00) = -9.00$$

that is, the acceleration at this time is $-9.00\,\frac{\mathrm{m}}{\mathrm{s}^2}$.

(d) From part (c) we note that at the time that the velocity was instantaneously zero the acceleration was *negative*. This means that the velocity was *decreasing* at the time. If the velocity was decreasing yet instantaneously equal to zero then it had to be going from positive to negative values at $t = 2.00\,\mathrm{s}$. So just before this time its velocity was positive.

(e) Likewise, from our answer to part (d) just after $t = 2.00\,\mathrm{s}$ the velocity of particle had to be negative.

(f) We have seen that the dot never gets to the right edge of the screen at $x = 15.0\,\mathrm{cm}$. It will not reverse its velocity again since $t = 2.00\,\mathrm{s}$ is the only positive time at which $v = 0$. So it will keep moving to back to the left, and the coordinate x will equal zero when we have:

$$x = 0 = 9.00t - 0.750t^3$$

Factor out t to solve:

$$t(9.00 - 0.750t^2) = 0 \qquad \Longrightarrow \qquad \begin{cases} t = 0 & \text{or} \\ (9.00 - 0.750t^2) = 0 & \text{otherwise.} \end{cases}$$

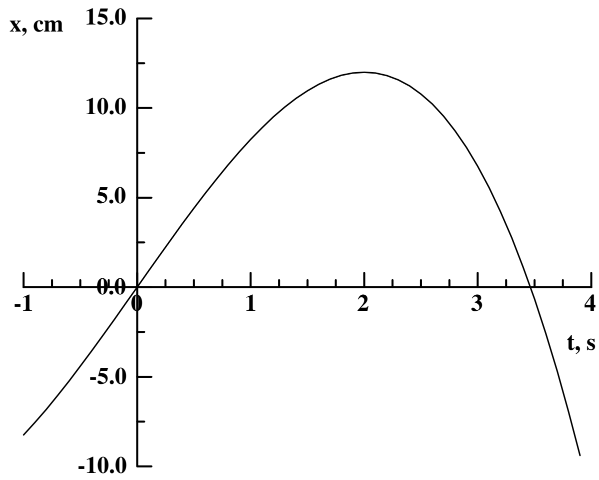

Figure 2.2: Plot of x vs t for moving spot. Ignore the parts where x is negative!

The first solution is the time that the dot started moving, so that is not the one we want. The second case gives:

$$(9.00 - 0.750t^2) = 0 \qquad \Longrightarrow \qquad t^2 = \frac{9.00}{0.750} = 12.0\,\mathrm{s}^2$$

which gives

$$t = 3.46\,\mathrm{s}$$

since we only want the positive solution. So the dot returns to $x = 0$ (the left side of the screen) at $t = 3.46\,\mathrm{s}$.

If we plot the original function $x(t)$ we get the curve given in Fig. 2.2 which shows that the spot does not get to $x = 15.0\,\mathrm{cm}$ before it turns around. (However as explained in the problem, the curve does *not* extend to negative values as the graph indicates.)

2.2.3 Constant Acceleration

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6. The head of a rattlesnake can accelerate $50\,\frac{\mathrm{m}}{\mathrm{s}^2}$ in striking a victim. If a car could do as well, how long would it take to reach a speed of $100\,\frac{\mathrm{km}}{\mathrm{hr}}$ from rest? [HRW5 2-33]

First, convert the car's final speed to SI units to make it easier to work with:

$$100\,\frac{\mathrm{km}}{\mathrm{hr}} = \left(100\,\frac{\mathrm{km}}{\mathrm{hr}}\right) \cdot \left(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}\right) \cdot \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right) = 27.8\,\frac{\mathrm{m}}{\mathrm{s}}$$

The acceleration of the car is $50\,\frac{\mathrm{m}}{\mathrm{s}^2}$ and it starts from rest which means that $v_0 = 0$. As we've found, the final velocity v of the car is $27.8\,\frac{\mathrm{m}}{\mathrm{s}}$. (The problem actually that this is final *speed* but if our coordinate system points in the same direction as the car's motion, these are the same thing.) Equation 2.6 lets us solve for the time t:

$$v = v_0 + at \qquad \Longrightarrow \qquad t = \frac{v - v_0}{a}$$

Substituting, we find

$$t = \frac{27.8\,\frac{\text{m}}{\text{s}} - 0}{50\,\frac{\text{m}}{\text{s}^2}} = 0.55\,\text{s}$$

If a car had such a large acceleration, it would take 0.55 s to attain the given speed.

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7. A body moving with uniform acceleration has a velocity of $12.0\,\frac{\text{cm}}{\text{s}}$ when its x coordinate is $3.00\,\text{cm}$. If its x coordinate $2.00\,\text{s}$ later is $-5.00\,\text{cm}$, what is the magnitude of its acceleration? [Ser4 2-25]

In this problem we are given the initial coordinate ($x = 3.00\,\text{cm}$), the initial velocity ($v_0 = 12.0\,\frac{\text{cm}}{\text{s}}$), the final x coordinate ($x = -5.00\,\text{cm}$) and the elapsed time (2.00 s). Using Eq. 2.7 (since we are told that the acceleration *is* constant) we can solve for a. We find:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \Longrightarrow \qquad \tfrac{1}{2}at^2 = x - x_0 - v_0 t$$

Substitute things:

$$\tfrac{1}{2}at^2 = -5.00\,\text{cm} - 3.00\,\text{cm} - \left(12.0\,\tfrac{\text{cm}}{\text{s}}\right)(2.00\,\text{s}) = -32.0\,\text{cm}$$

Solve for a:

$$a = \frac{2(-32.0\,\text{cm})}{t^2} = \frac{2(-32.0\,\text{cm})}{(2.00\,\text{s})^2} = -16.0\frac{\text{cm}}{\text{s}^2}$$

The x acceleration of the object is $-16.\,\frac{\text{cm}}{\text{s}^2}$. (The magnitude of the acceleration is $16.0\,\frac{\text{cm}}{\text{s}^2}$.)

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8. A jet plane lands with a velocity of $100\,\frac{\text{m}}{\text{s}}$ and can accelerate at a maximum rate of $-5.0\,\frac{\text{m}}{\text{s}^2}$ as it comes to rest. (a) From the instant it touches the runway, what is the minimum time needed before it stops? (b) Can this plane land at a small airport where the runway is $0.80\,\text{km}$ long? [Ser4 2-31]

(a) The data given in the problem is illustrated in Fig. 2.3. The minus sign in the acceleration indicates that the sense of the acceleration is opposite that of the motion, that is, the plane is decelerating.

The plane will stop as quickly as possible if the acceleration *does* have the value $-5.0\,\frac{\text{m}}{\text{s}^2}$, so we use this value in finding the time t in which the velocity changes from $v_0 = 100\,\frac{\text{m}}{\text{s}}$ to $v = 0$. Eq. 2.6 tells us:

$$t = \frac{v - v_0}{a}$$

Substituting, we find:

$$t = \frac{(0 - 100\,\frac{\text{m}}{\text{s}})}{(-5.0\,\frac{\text{m}}{\text{s}^2})} = 20\,\text{s}$$

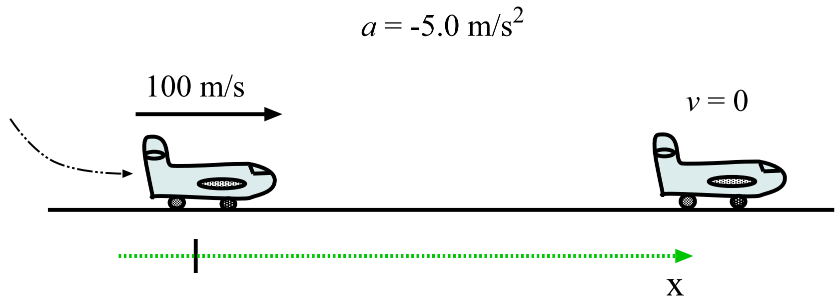

Figure 2.3: Plane touches down on runway at $100\,\frac{\text{m}}{\text{s}}$ and comes to a halt.

The plane needs 20 s to come to a halt.

(b) The plane also travels the shortest distance in stopping if its acceleration is $-5.0\,\frac{\text{m}}{\text{s}^2}$. With $x_0 = 0$, we can find the plane's final x coordinate using Eq. 2.9, using $t = 20\,\text{s}$ which we got from part (a):

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = 0 + \tfrac{1}{2}(100\,\tfrac{\text{m}}{\text{s}} + 0)(20\,\text{s}) = 1000\,\text{m} = 1.0\,\text{km}$$

The plane must have at least 1.0 km of runway in order to come to a halt safely. 0.80 km is *not* sufficient.

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9. A drag racer starts her car from rest and accelerates at $10.0\,\frac{\text{m}}{\text{s}^2}$ for the entire distance of $400\,\text{m}$ ($\frac{1}{4}$ mile). (a) How long did it take the car to travel this distance? (b) What is the speed at the end of the run? [Ser4 2-33]

(a) The racer moves in one dimension (along the x axis, say) with constant acceleration $a = 10.0\,\frac{\text{m}}{\text{s}^2}$. We can take her initial coordinate to be $x_0 = 0$; she starts from rest, so that $v_0 = 0$. Then the location of the car (x) is given by:

$$\begin{aligned} x &= x_0 + v_0 t + \tfrac{1}{2}at^2 \\ &= = 0 + 0 + \tfrac{1}{2}at^2 = \tfrac{1}{2}(10.0\,\tfrac{\text{m}}{\text{s}^2})t^2 \end{aligned}$$

We want to know the time at which $x = 400\,\text{m}$. Substitute and solve for t:

$$400\,\text{m} = \tfrac{1}{2}(10.0\,\tfrac{\text{m}}{\text{s}^2})t^2 \qquad \Longrightarrow \qquad t^2 = \frac{2(400\,\text{m})}{(10.0\,\frac{\text{m}}{\text{s}^2})} = 80.0\,\text{s}^2$$

which gives

$$t = 8.94\,\text{s}\ .$$

The car takes 8.94 s to travel this distance.

(b) We would like to find the velocity at the end of the run, namely at $t = 8.94\,\text{s}$ (the time we found in part (a)). The velocity is:

$$\begin{aligned} v &= v_0 + at \\ &= 0 + (10.0\,\tfrac{\text{m}}{\text{s}^2})t = (10.0\,\tfrac{\text{m}}{\text{s}^2})t \end{aligned}$$

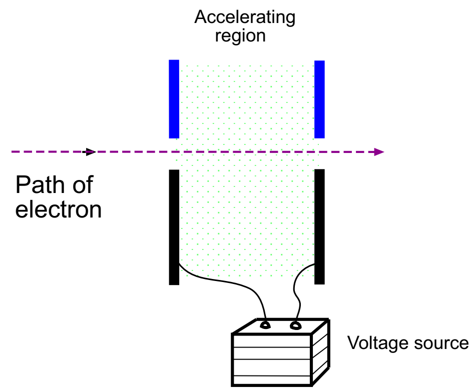

Figure 2.4: Electron is accelerated in a region between two plates, in Example 10.

At $t = 8.94\,\mathrm{s}$, the velocity is

$$v = (10.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(8.94\,\mathrm{s}) = 89.4\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

The speed at the end of the run is $89.4\,\frac{\mathrm{m}}{\mathrm{s}}$.

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10. An electron with initial velocity $v_0 = 1.50 \times 10^5\,\frac{\mathrm{m}}{\mathrm{s}}$ enters a region $1.0\,\mathrm{cm}$ long where it is electrically accelerated, as shown in Fig. 2.4. It emerges with velocity $v = 5.70 \times 10^6\,\frac{\mathrm{m}}{\mathrm{s}}$. What was its acceleration, assumed constant? (Such a process occurs in the electron gun in a cathode–ray tube, used in television receivers and oscilloscopes.) [HRW5 2-39]

We are told that the acceleration of the electron is constant, so that Eqs. 2.6–2.9 can be used.

Here we know the initial and final velocities of the electron (v_0 and v). If we let its initial coordinate be $x_0 = 0$ then the final coordinate is $x = 1.0\,\mathrm{cm} = 1.0 \times 10^{-2}\,\mathrm{m}$. We don't know the time t for its travel through the accelerating region and of course we don't know the (constant) acceleration, which is what we're being asked in this problem.

We see that we can solve for a if we use Eq. 2.8:

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \Longrightarrow \qquad a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

Substitute and get:

$$\begin{aligned} a &= \frac{(5.70 \times 10^6\,\frac{\mathrm{m}}{\mathrm{s}})^2 - (1.50 \times 10^5\,\frac{\mathrm{m}}{\mathrm{s}})^2}{2(1.0 \times 10^{-2}\,\mathrm{m})} \\ &= 1.62 \times 10^{15}\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \end{aligned}$$

The acceleration of the electron is $1.62 \times 10^{15}\,\frac{\mathrm{m}}{\mathrm{s}^2}$ (while it is in the accelerating region).

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11. A world's land speed record was set by Colonel John P. Stapp when on March 19, 1954 he rode a rocket–propelled sled that moved down a track at $1020\,\frac{\text{km}}{\text{h}}$. He and the sled were brought to a stop in $1.4\,\text{s}$. What acceleration did he experience? Express your answer in g units. [HRW5 2-41]

For the period of deceleration of the rocket sled (which lasts for 1.4 s) were are given the initial velocity and the final velocity, which is *zero* since the sled comes to rest at the end.

First, convert his initial velocity to SI units:

$$v_0 = 1020\,\frac{\text{km}}{\text{h}} = (1020\,\frac{\text{km}}{\text{h}})\left(\frac{10^3\,\text{m}}{1\,\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 283.3\,\frac{\text{m}}{\text{s}}$$

The Eq. 2.6 gives us the acceleration a:

$$v = v_0 + at \qquad \Longrightarrow \qquad a = \frac{v - v_0}{t}$$

Substitute:

$$a = \frac{0 - 283.3\,\frac{\text{m}}{\text{s}}}{1.4\,\text{s}} = -202.4\,\frac{\text{m}}{\text{s}^2}$$

The acceleration is a *negative* number since it is opposite to the sense of the motion; it is a *deceleration*. The *magnitude* of the sled's acceleration is $202.4\,\frac{\text{m}}{\text{s}^2}$.

To express this as a multiple of g, we note that

$$\frac{|a|}{g} = \frac{202.4\,\frac{\text{m}}{\text{s}^2}}{9.80\,\frac{\text{m}}{\text{s}^2}} = 20.7$$

so the magnitude of the acceleration was $|a| = 20.7\,g$. That's a lotta g's!

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12. A subway train is traveling at $80\,\frac{\text{km}}{\text{h}}$ when it approaches a slower train $50\,\text{m}$ ahead traveling in the same direction at $25\,\frac{\text{km}}{\text{h}}$. If the faster train begins decelerating at $2.1\,\frac{\text{m}}{\text{s}^2}$ while the slower train continues at constant speed, how soon and at what relative speed will they collide? [wolf 2-73]

First, convert the initial speeds of the trains to units of $\frac{\text{m}}{\text{s}}$. We find:

$$80\,\frac{\text{km}}{\text{h}} = 22.2\,\frac{\text{m}}{\text{s}} \qquad\qquad 25\,\frac{\text{km}}{\text{h}} = 6.94\,\frac{\text{m}}{\text{s}}\ .$$

The situation of the trains at $t = 0$ (when the rear train begins to decelerate) is shown in Fig. 2.5. We choose the origin of the x axis to be at the initial position of the rear train; then the initial position of the front train is $x = 50\,\text{m}$. If we call the x–coordinate of the rear train x_1, then since it has initial velocity $22.2\,\frac{\text{m}}{\text{s}}$ and acceleration $-2.1\,\frac{\text{m}}{\text{s}^2}$ (note the minus sign!) the equation for $x_1(t)$ is

$$x_1(t) = (22.2\,\tfrac{\text{m}}{\text{s}})t + \tfrac{1}{2}(-2.1\,\tfrac{\text{m}}{\text{s}^2})t^2 = (22.2\,\tfrac{\text{m}}{\text{s}})t + (-1.05\,\tfrac{\text{m}}{\text{s}^2})t^2$$

Meanwhile, the front car has an initial velocity of $6.94\,\frac{\text{m}}{\text{s}}$ and *no* acceleration, so its coordinate (x_2) is given by

$$x_2(t) = 50\,\text{m} + (6.94\,\tfrac{\text{m}}{\text{s}})t$$

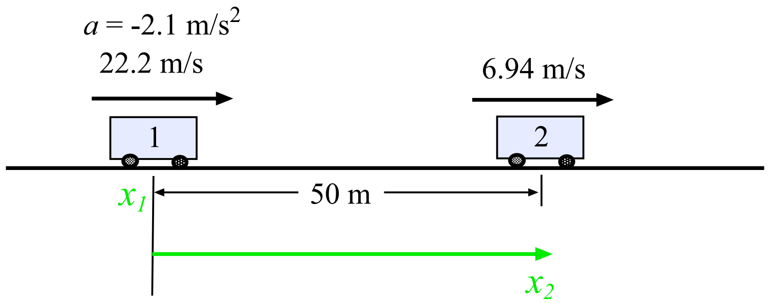

Figure 2.5: Two subway trains in Example 12.

The trains will collide if there is ever a time at which their coordinates are *equal*. So we want to see if there is a t which gives the condition:

$$(22.2\,\tfrac{\mathrm{m}}{\mathrm{s}})t + (-1.05\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t^2 = 50\,\mathrm{m} + (6.94\,\tfrac{\mathrm{m}}{\mathrm{s}})t$$

This is a quadratic equation, for which we can use the quadratic formula. Neglecting the units for simplicity, we can rearrange the terms and rewrite it as

$$1.05t^2 - 15.28t + 50 = 0$$

and the quadratic formula gives the answers as

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{15.28 \pm \sqrt{(15.28)^2 - 4(1.05)(50)}}{2(1.05)} = \begin{cases} 9.58\,\mathrm{s} \\ 4.97\,\mathrm{s} \end{cases}$$

This is a little confusing because there are *two* possible answers! (Both values of t are positive.) But the answer we want is the first one, 4.97 s — after the collision, the second time is not relevant[1]. So the trains will collide $t = 4.97\,\mathrm{s}$ after the rear car begins to decelerate.

At the time we have found, the velocity of the rear train is

$$v = v_0 + at = 22.2\,\tfrac{\mathrm{m}}{\mathrm{s}} + (-2.1\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(4.97\,\mathrm{s}) = 11.8\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

and the velocity of the front train remains $6.94\,\frac{\mathrm{m}}{\mathrm{s}}$. So at the time of the collision, the rear train is going faster by a difference of

$$\Delta v = 11.8\,\tfrac{\mathrm{m}}{\mathrm{s}} - 6.94\,\tfrac{\mathrm{m}}{\mathrm{s}} = 4.8\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

That is the *relative* speed at which the collision takes place.

2.2.4 Free Fall

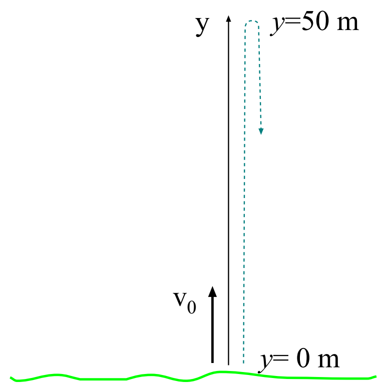

Figure 2.6: Object thrown upward reaches height of 50 m.

13. (a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50 m? (b) How long will it be in the air? [HRW5 2-61]

(a) First, we decide on a coordinate system. I will use the one shown in Fig. 2.6, where the y axis points upward and the origin is at ground level. The ball starts its flight from ground level so its initial position is $y_0 = 0$. When the ball is at maximum height its coordinate is $y = 50\,\mathrm{m}$, but *we also know its velocity at this point*. At maximum height the instantaneous velocity of the ball is *zero*. So if our "final" point is the time of maximum height, then $v = 0$.

So for the trip from ground level to maximum height, we know y_0, y, v and the acceleration $a = -9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = -g$, but we *don't* know v_0 or the time t to get to maximum height.

From our list of constant–acceleration equations, we see that Equation 2.8 will give us the initial velocity v_0:

$$v^2 = v_0^2 + 2a(y - y_0) \qquad \Longrightarrow \qquad v_0^2 = v^2 - 2a(y - y_0)$$

Substitute, and get:

$$v_0^2 = (0)^2 - 2(-9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(50\,\mathrm{m} - 0) = 980\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

The next step is to "take the square root". Since we know that v_0 must be a *positive* number, we know that we should take the positive square root of $980\,\frac{\mathrm{m}^2}{\mathrm{s}^2}$. We get:

$$v_0 = +31\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

The initial speed of the ball is $31\,\frac{\mathrm{m}}{\mathrm{s}}$

(b) We want to find the total time that the ball is in flight. What do we know about the ball when it returns to earth and hits the ground? *We know that its y coordinate is equal to zero.* (So far, we don't know anything about the ball's *velocity* at the the time it returns to ground level.) If we consider the time between throwing and impact, then we do know y_0, y, v_0 and of course a. If we substitute into Eq. 2.7 we find:

[1]However it *would* be relevant if the trains were on parallel tracks; then the collision would not take place and we could find the *times* at which they were side-by-side and their relative velocities at those times.

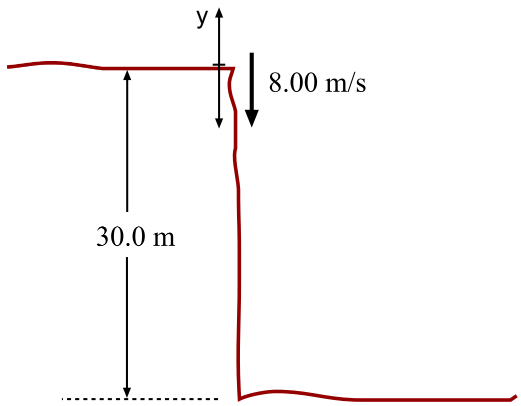

Figure 2.7: Ball is thrown straight down with speed of $8.00\,\frac{\mathrm{m}}{\mathrm{s}}$, in Example 14.

$$0 = 0 + (31\,\tfrac{\mathrm{m}}{\mathrm{s}})t + \tfrac{1}{2}(-9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t^2$$

It is not hard to solve this equation for t. We can factor it to give:

$$t[(31\,\tfrac{\mathrm{m}}{\mathrm{s}}) + \tfrac{1}{2}(-9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t] = 0$$

which has two solutions. One of them is simply $t = 0$. This solution *is* an answer to the question we are asking, namely "When does $y = 0$?" because the ball was at ground level at $t = 0$. But it is not the solution we want. For the other solution, we must have:

$$(31\,\tfrac{\mathrm{m}}{\mathrm{s}}) + \tfrac{1}{2}(-9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t = 0$$

which gives

$$t = \frac{2(31\,\frac{\mathrm{m}}{\mathrm{s}})}{9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 6.4\,\mathrm{s}$$

The ball spends a total of 6.4 seconds in flight.

---

14. A ball is thrown directly downward with an initial speed of $8.00\,\frac{\mathrm{m}}{\mathrm{s}}$ from a height of $30.0\,\mathrm{m}$. When does the ball strike the ground? [Ser4 2-46]

We diagram the problem as in Fig. 2.7. We have to choose a coordinate system, and here I will put the let the origin of the y axis be at the place where the ball starts its motion (at the top of the 30 m height). With this choice, the ball starts its motion at $y = 0$ and strikes the ground when $y = -30\,\mathrm{m}$.

We can now see that the problem is asking us: At what time does $y = -30.0\,\mathrm{m}$? We have $v_0 = -8.00\,\frac{\mathrm{m}}{\mathrm{s}}$ (*minus* because the ball is thrown *downward*!) and the acceleration of the the ball is $a = -g = -9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$, so at any time t the y coordinate is given by

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = (-8.00\,\tfrac{\mathrm{m}}{\mathrm{s}})t - \tfrac{1}{2}gt^2$$

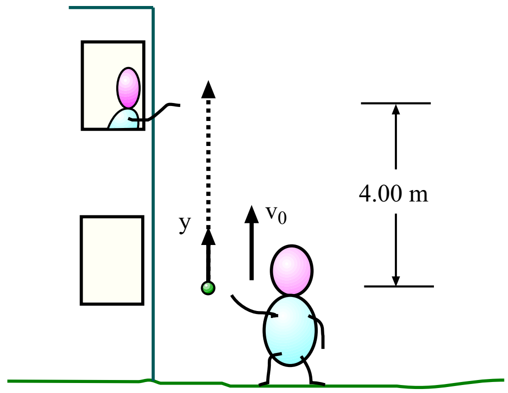

Figure 2.8: Student throws her keys into the air, in Example 15.

But at the time of impact we have

$$y = -30.0\,\mathrm{m} = (-8.00\,\tfrac{\mathrm{m}}{\mathrm{s}})t - \tfrac{1}{2}gt^2 = (-8.00\,\tfrac{\mathrm{m}}{\mathrm{s}})t - (4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t^2\ ,$$

an equation for which we can solve for t. We rewrite it as:

$$(4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t^2 + (8.00\,\tfrac{\mathrm{m}}{\mathrm{s}})t - 30.0\,\mathrm{m} = 0$$

which is just a quadratic equation in t. From our algebra courses we know how to solve this; the solutions are:

$$t = \frac{-(8.00\,\tfrac{\mathrm{m}}{\mathrm{s}}) \pm \sqrt{(8.00\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 - 4(4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(-30.0\,\mathrm{m})}}{2(4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})}$$

and a little calculator work finally gives us:

$$t = \begin{cases} -3.42\,\mathrm{s} \\ 1.78\,\mathrm{s} \end{cases}$$

Our answer is one of these . . . which one? Obviously the ball had to strike the ground at some *positive* value of t, so the answer is $t = 1.78\,\mathrm{s}$.

The ball strikes the ground 1.78 s after being thrown.

---

15. A student throws a set of keys vertically upward to her sorority sister in a window 4.00 m **above. The keys are caught** 1.50 s **later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?** [Ser4 2-47]

(a) We draw a simple picture of the problem; such a simple picture is given in Fig. 2.8. Having a picture is important, but we should be careful not to put *too much* into the picture; the problem did not say that the keys were caught while they were going up or going down. For all we know at the moment, it could be either one!

We will put the origin of the y axis at the point where the keys were thrown. This simplifies things in that the initial y coordinate of the keys is $y_0 = 0$. Of course, since this is a problem about free–fall, we know the acceleration: $a = -g = -9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$.

What mathematical information does the problem give us? We are told that when $t = 1.50\,\mathrm{s}$, the y coordinate of the keys is $y = 4.00\,\mathrm{m}$. Is this enough information to solve the problem? We write the equation for $y(t)$:

$$\begin{aligned} y &= y_0 + v_0 t + \tfrac{1}{2}at^2 \\ &= v_0 t - \tfrac{1}{2}gt^2 \end{aligned}$$

where v_0 is presently unknown. At $t = 1.50\,\mathrm{s}$, $y = 4.00\,\mathrm{m}$, so:

$$4.00\,\mathrm{m} = v_0(1.50\,\mathrm{s}) - \tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.50\,\mathrm{s})^2 \ .$$

Now we can solve for v_0. Rearrange this equation to get:

$$v_0(1.50\,\mathrm{s}) = 4.00\,\mathrm{m} + \tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.50\,\mathrm{s})^2 = 15.0\,\mathrm{m} \ .$$

So:

$$v_0 = \frac{15.0\,\mathrm{m}}{1.50\,\mathrm{s}} = 10.0\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

(b) We want to find the velocity of the keys at the time they were caught, that is, at $t = 1.50\,\mathrm{s}$. We know v_0; the velocity of the keys at all times follows from Eq. 2.6,

$$v = v_0 + at = 10.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - 9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}t$$

So at $t = 1.50\,\mathrm{s}$,

$$v = 10.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - 9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}(1.50\,\mathrm{s}) = -4.68\,\tfrac{\mathrm{m}}{\mathrm{s}} \ .$$

So the velocity of the keys when they were caught was $\boxed{-4.68\,\frac{\mathrm{m}}{\mathrm{s}}}$. Note that the keys had a *negative* velocity; this tells us that the keys were moving *downward* at the time they were caught!

---

16. A ball is thrown vertically upward from the ground with an initial speed of $15.0\,\frac{\mathrm{m}}{\mathrm{s}}$. (a) How long does it take the ball to reach its maximum altitude? (b) What is its maximum altitude? (c) Determine the velocity and acceleration of the ball at $t = 2.00\,\mathrm{s}$. [Ser4 2-49]

(a) An illustration of the data given in this problem is given in Fig. 2.9. We measure the coordinate y upward from the place where the ball is thrown so that $y_0 = 0$. The ball's acceleration while in flight is $a = -g = -9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$. We are given that $v_0 = +15.0\,\frac{\mathrm{m}}{\mathrm{s}}$.

The ball is at maximum altitude when its (instantaneous) velocity v is *zero* (it is neither going up nor going down) and we can use the expression for v to solve for t:

$$v = v_0 + at \qquad \Longrightarrow \qquad t = \frac{v - v_0}{a}$$

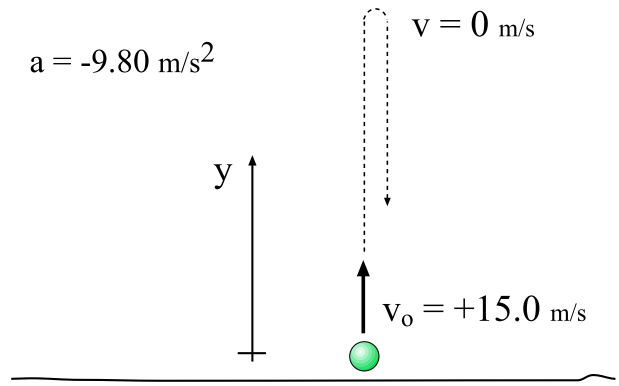

Figure 2.9: Ball is thrown straight up with initial speed $15.0\,\frac{\mathrm{m}}{\mathrm{s}}$.

Plug in the values for the *top* of the ball's flight and get:

$$t = \frac{(0) - (15.0\,\frac{\mathrm{m}}{\mathrm{s}})}{(-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 1.53\,\mathrm{s}\ .$$

The ball takes $\boxed{1.53\,\mathrm{s}}$ to reach maximum height.

(b) Now that we have the value of t when the ball is at maximum height we can plug it into Eq. 2.7 and find the value of y at this time and that will be the *value* of the maximum height. But we can also use Eq. 2.8 since we know all the values except for y. Solving for y we find:

$$v^2 = v_0^2 + 2ay \qquad \Longrightarrow \qquad y = \frac{v^2 - v_0^2}{2a}$$

Plugging in the numbers, we get

$$y = \frac{(0)^2 - (15.0\,\frac{\mathrm{m}}{\mathrm{s}})^2}{2(-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 11.5\,\mathrm{m}$$

The ball reaches a maximum height of $\boxed{11.5\,\mathrm{m}}$.

(c) At $t = 2.00\,\mathrm{s}$ (that is, 2.0 seconds after the ball was thrown) we use Eq. 2.6 to find:

$$v = v_0 + at = (15.0\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (-9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(2.00\,\mathrm{s}) = -4.60\,\tfrac{\mathrm{m}}{\mathrm{s}}\ .$$

so at $t = 2.00\,\mathrm{s}$ the ball is on its way back *down* with a speed of $\boxed{4.60\,\frac{\mathrm{m}}{\mathrm{s}}}$.

As for the next part, the acceleration of the ball is *always* equal to $\boxed{-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}}$ while it is in flight.

---

17. A baseball is hit such that it travels straight upward after being struck by the bat. A fan observes that it requires $3.00\,\mathrm{s}$ for the ball to reach its maximum height. Find (a) its initial velocity and (b) its maximum height. Ignore the effects of air resistance. [Ser4 2-51]

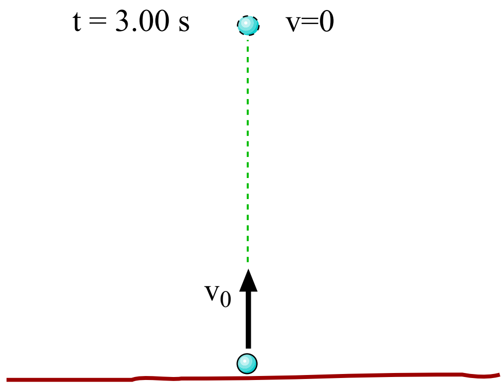

Figure 2.10: Ball is hit straight up; reaches maximum height 3.00 s later.

(a) An illustration of the data given in the problem is given in Fig. 2.10.

For the period from when the ball is hit to the time it reaches maximum height, we know the time interval, the acceleration ($a = -g$) and also the final velocity, since at maximum height the velocity of the ball is *zero*. Then Eq. 2.6 gives us v_0:

$$v = v_0 + at \qquad \Longrightarrow \qquad v_0 = v - at$$

and we get:

$$v_0 = 0 - (-9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(3.00\,\mathrm{s}) = 29.4\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

The initial velocity of the ball was $\boxed{+29.4\,\frac{\mathrm{m}}{\mathrm{s}}}$.

(b) To find the value of the maximum height, we need to find the value of the y coordinate at time $t = 3.00\,\mathrm{s}$. We can use either Eq. 2.7 or Eq. 2.8. the latter gives:

$$v^2 = v_0^2 + 2a(y - y_0) \qquad \Longrightarrow \qquad (y - y_0) = \frac{v^2 - v_0^2}{2a}$$

Plugging in the numbers we find that the change in y coordinate for the trip up was:

$$y - y_0 = \frac{0^2 - (29.4\,\frac{\mathrm{m}}{\mathrm{s}})^2}{2(-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 44.1\,\mathrm{m}\ .$$

The ball reached a maximum height of $\boxed{44.1\,\mathrm{m}}$.

---

18. A parachutist bails out and freely falls $50\,\mathrm{m}$. Then the parachute opens, and thereafter she decelerates at $2.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$. She reaches the ground with a speed of $3.0\,\frac{\mathrm{m}}{\mathrm{s}}$. (a) How long was the parachutist in the air? (b) At what height did the fall begin? [HRW5 2-84]

(a) This problem gives several odd bits of information about the motion of the parachutist! We organize the information by *drawing a diagram*, like the one given in Fig. 2.11. It is

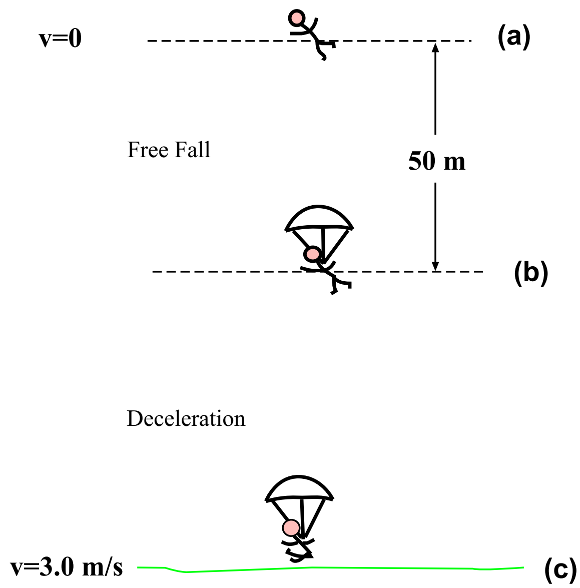

Figure 2.11: Diagram showing motion of parachutist in Example 18.

very important to organize our work in this way!

At the height indicated by (a) in the figure, the skydiver has *zero* initial speed. As she falls from (a) to (b) her acceleration is that of gravity, namely $9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$ *downward*. We know that (b) is 50 m lower than (a) but we don't yet know the skydiver's speed at (b). Finally, at point (c) her speed is $3.0\,\frac{\mathrm{m}}{\mathrm{s}}$ and between (b) and (c) her "deceleration" was $2.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$, but we don't know the difference in height between (b) and (c).

How can we start to fill in the gaps in our knowledge?

We note that on the trip from (a) to (b) we do know the starting velocity, the distance travelled and the acceleration. From Eq. 2.8 we can see that this is enough to find the final velocity, that is, the velocity at (b).

Use a coordinate system (y) which has its origin at level (b), and the y axis pointing upward. Then the initial y coordinate is $y_0 = 50\,\mathrm{m}$ and the the initial velocity is $v_0 = 0$. The final y coordinate is $y = 0$ and the acceleration is constant at $a = -9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Then using Eq. 2.8 we have:

$$v^2 = v_0^2 + 2a(y - y + 0) = 0 + 2(-9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(0 - 50\,\mathrm{m}) = 980\,\tfrac{\mathrm{m}^2}{\mathrm{s}^2}$$

which has the solutions

$$v = \pm 31.3\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

but here the skydiver is obviously moving *downward* at (b), so we must pick

$$v = -31.3\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

for the velocity at (b).

While we're at it, we can find the time it took to get from (a) to (b) using Eq. 2.6, since we know the velocities and the acceleration for the motion. We find:

$$v = v_0 + at \qquad \Longrightarrow \qquad t = \frac{v - v_0}{a}$$

Substitute:

$$t = \frac{(-31.3\,\frac{\rm m}{\rm s} - 0)}{-9.80\,\frac{\rm m}{\rm s^2}} = 3.19\,{\rm s}$$

The skydiver takes 3.19 s to fall from (a) to (b).

Now we consider the motion from (b) to (c). For this part of the motion we know the initial and final velocities. We also know the acceleration, but we must be careful about how it is expressed. During this part of the trip, the skydiver's motion is always downward (velocity is always negative) but her speed decreases from $31.9\,\frac{\rm m}{\rm s}$ to $3.0\,\frac{\rm m}{\rm s}$. The *velocity* changes from $-31.3\,\frac{\rm m}{\rm s}$ to $-3.0\,\frac{\rm m}{\rm s}$ so that the velocity has *increased*. The acceleration is *positive*; it is in the opposite sense as the motion and thus it was rightly called a "deceleration" in the problem. So for the motion from (b) to (c), we have

$$a = +2.0\,\frac{\rm m}{\rm s^2}$$

We have the starting and final velocities for the trip from (b) to (c) so Eq. 2.6 lets us solve for the time t:

$$v = v_0 + at \qquad \Longrightarrow \qquad t = \frac{v - v_0}{a}$$

Substitute:

$$t = \frac{-3.0\,\frac{\rm m}{\rm s} - (-31.3\,\frac{\rm m}{\rm s})}{+2.0\,\frac{\rm m}{\rm s^2}} = 14.2\,{\rm s}$$

Now we are prepared to answer part (a) of the problem. The time of the travel from (a) was 3.19 s; the time of travel from (b) to (c) was 14.2 s. The total time in the air was

$$t_{\rm Total} = 3.19\,{\rm s} + 14.2\,{\rm s} = 17.4\,{\rm s}$$

(b) Let's keep thinking about the trip from (b) to (c); we'll keep the origin at the same place as before (at (b)). Then for the trip from (b) to (c) the initial coordinate is $y_0 = 0$. The initial velocity is $v_0 = -31.9\,\frac{\rm m}{\rm s}$ and the final velocity is $v = -3.0\,\frac{\rm m}{\rm s}$. We have the acceleration, so Eq. 2.8 gives us the final coordinate y:

$$v^2 = v_0^2 + 2a(y - y_0) \qquad \Longrightarrow \qquad y - y_0 = \frac{v^2 - v_0^2}{2a}$$

Substitute:

$$y - y_0 = \frac{(-3.0\,\frac{\rm m}{\rm s})^2 - (-31.3\,\frac{\rm m}{\rm s})^2}{2(+2.0\,\frac{\rm m}{\rm s^2})} = -243\,{\rm m}$$

Since we chose $y_0 = 0$, the final coordinate of the skydiver is $y = -243\,{\rm m}$.

We have used the same coordinate system in both parts, so overall the skydiver has gone from $y = +50\,{\rm m}$ to $y = -243\,{\rm m}$. The change in height was

$$\Delta y = -243\,{\rm m} - 50\,{\rm m} = -293\,{\rm m}$$

So the parachutist's fall began at a height of $\boxed{293\,{\rm m}}$ above the ground.

19. A stone falls from rest from the top of a high cliff. a second stone is thrown downward from the same height $2.00\,\mathrm{s}$ later with an initial speed of $30.0\,\frac{\mathrm{m}}{\mathrm{s}}$. If both stones hit the ground simultaneously, how high is the cliff? [Ser4 2-54]

This is a "puzzle"–type problem which goes beyond the normal substitute–and–solve type; it involves more organization of our work and a clear understanding of our equations. Here's the way I would attack it.

We have two different falling objects here with their own coordinates; we'll put our origin at the top of the cliff and call the y coordinate of the first stone y_1 and that of the second stone y_2. Each has a different dependence on the time t.

For the first rock, we have $v_0 = 0$ since it falls from rest and of course $a = -g$ so that its position is given by

$$y_1 = y_0 + v_0 t + \tfrac{1}{2}at^2 = -\tfrac{1}{2}gt^2$$

This is simple enough but we need to remind ourselves that here t is the time since the *first* stone started its motion. It is *not* the same as the time since the *second* stone starts its motion. To be clear, let's call this time t_1. So we have:

$$y_1 = -\tfrac{1}{2}gt_1^2 = -4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2}t_1^2$$

Now, for the motion of the second stone, if we write t_2 for the time since *it* started its motion, the facts stated in the problem tell us that its y coordinate is given by:

$$y_2 = y_0 + v_0 t_2 + \tfrac{1}{2}at_2^2 = (-30.0\,\tfrac{\mathrm{m}}{\mathrm{s}})t_2 - \tfrac{1}{2}gt_2^2$$

So far, so good. The problem tells us that the first stone has been falling for $2.0\,\mathrm{s}$ longer than the second one. This means that t_1 is $2.0\,\mathrm{s}$ larger than t_2. So:

$$t_1 = t_2 + 2.0\,\mathrm{s} \qquad \Longrightarrow \qquad t_2 = t_1 - 2.0\,\mathrm{s}$$

(We will use t_1 as our one time variable.) Putting this into our last equation and doing some algebra gives

$$\begin{aligned} y_2 &= (-30.0\,\tfrac{\mathrm{m}}{\mathrm{s}})(t_1 - 2.0\,\mathrm{s}) - \tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(t_1 - 2.0\,\mathrm{s})^2 \\ &= (-30.0\,\tfrac{\mathrm{m}}{\mathrm{s}})(t_1 - 2.0\,\mathrm{s}) - (4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(t_1^2 - 4.0\,\mathrm{s}t_1 + 4.0\,\mathrm{s}^2)^2 \\ &= (-4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 + (-30.0\,\tfrac{\mathrm{m}}{\mathrm{s}} + 19.6\,\tfrac{\mathrm{m}}{\mathrm{s}})t_1 + (60.0\,\mathrm{m} - 19.6\,\mathrm{m}) \\ &= (-4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 + (-10.4\,\tfrac{\mathrm{m}}{\mathrm{s}})t_1 + (40.4\,\mathrm{m}) \end{aligned}$$

We need to remember that this expression for y_2 will be meaningless for values of t_1 which are less than $2.0\,\mathrm{s}$. With this expression we can find values of y_1 and y_2 using the *same* time coordinate, t_1.

Now, the problem tells us that at some time (t_1) the coordinates of the two stones are *equal*. We don't yet yet know what that time or coordinate *is* but that is the information contained in the statement "both stones hit the ground simultaneously". We can find this time by setting y_1 equal to y_2 and solving:

$$(-4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 = (-4.90\,\tfrac{\mathrm{m}}{\mathrm{s}^2})t_1^2 + (-10.4\,\tfrac{\mathrm{m}}{\mathrm{s}})t_1 + (40.4\,\mathrm{m})$$

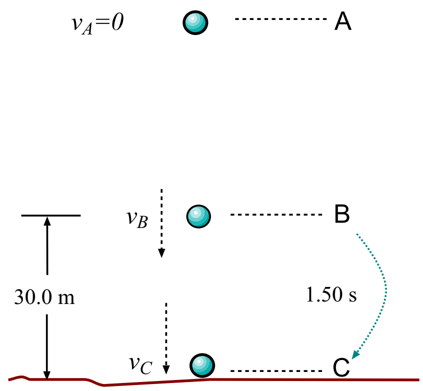

Figure 2.12: Diagram for the falling object in Example 20.

Fortunately the t^2 term cancels in this equation making it a lot easier. We get:

$$(-10.4\,\frac{\mathrm{m}}{\mathrm{s}})t_1 + (40.4\,\mathrm{m}) = 0$$

which has the solution

$$t_1 = \frac{40.4\,\mathrm{m}}{10.4\,\frac{\mathrm{m}}{\mathrm{s}}} = 3.88\,\mathrm{s}$$

So the rocks will have the same location at $t_1 = 3.88\,\mathrm{s}$, that is, $3.88\,\mathrm{s}$ after the first rock has been dropped.

What *is* that location? We can find this by using our value of t_1 to get either y_1 or y_2 (the answer will be the same). Putting it into the expression for y_1 we get:

$$y_1 = -4.90\,\frac{\mathrm{m}}{\mathrm{s}^2}t_1^2 = (-4.90\,\frac{\mathrm{m}}{\mathrm{s}^2})(3.88\,\mathrm{s})^2 = -74\,\mathrm{m}$$

So *both* stones were 74 m below the initial point at the time of impact. The cliff is $\boxed{74\,\mathrm{m}}$ high.

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20. A falling object requires $1.50\,\mathrm{s}$ **to travel the last** $30.0\,\mathrm{m}$ **before hitting the ground. From what height above the ground did it fall?** [Ser4 2-68]

This is an intriguing sort of problem... very easy to state, but not so clear as to where to begin in setting it up!

The first thing to do is *draw a diagram*. We draw the important points of the object's motion, as in Fig. 2.12. The object has zero velocity at A; at B it is at a height of $30.0\,\mathrm{m}$ above the ground with an unknown velocity. At C it is at ground level, the time is $1.50\,\mathrm{s}$ later than at B and we also don't know the velocity here. Of course, we know the acceleration: $a = -9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}$!!

We are given all the information about the trip from B to C, so why not try to fill in our knowledge about this part? We know the final and initial coordinates, the acceleration and the time so we can find the initial velocity (that is, the velocity at B). Let's put the origin at ground level; then, $y_0 = 30.0\,\mathrm{m}$, $y = 0$ and $t = 1.50\,\mathrm{s}$, and using

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

we find:

$$v_0 t = (y - y_0) - \tfrac{1}{2}at^2 = (0 - (30.0\,\mathrm{m})) - \tfrac{1}{2}(-9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.50)^2 = -19.0\,\mathrm{m}$$

so that

$$v_0 = \frac{(-19.0\,\mathrm{m})}{t} = \frac{(-19.0\,\mathrm{m})}{(1.50\,\mathrm{s})} = -12.5\,\tfrac{\mathrm{m}}{\mathrm{s}}\ .$$

This is the velocity at point B; we can also find the velocity at C easily, since that is the final velocity, v:

$$v = v_0 + at = (-12.5\,\tfrac{\mathrm{m}}{\mathrm{s}}) + (-9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.50\,\mathrm{s}) = -27.3\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Now we can consider the trip from the starting point, A to the point of impact, C. We don't know the initial y coordinate, but we *do* know the final and initial velocities: The initial velocity is $v_0 = 0$ and the final velocity is $v = -27.3\,\frac{\mathrm{m}}{\mathrm{s}}$, as we just found. With the origin set at ground level, the final y coordinate is $y = 0$. We don't know the *time* for the trip, but if we use:

$$v^2 = v_0^2 + 2a(y - y_0)$$

we find:

$$(y - y_0) = \frac{(v^2 - v_0^2)}{2a} = \frac{(-27.3\,\frac{\mathrm{m}}{\mathrm{s}})^2 - (0)^2}{2(-9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = -38.2\,\mathrm{m}$$

and we can rearrange this to get:

$$y_0 = y + 38.2\,\mathrm{m} = 0 + 38.2\,\mathrm{m} = 38.2\,\mathrm{m}$$

and the so the object started falling from a height of $\boxed{38.2\,\mathrm{m}}$.

There are probably cleverer ways to do this problem, but here I wanted to give you the slow, patient approach!

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21. A student is staring idly out her dormitory window when she sees a water balloon fall past. If the balloon takes $0.22\,\mathrm{s}$ to cross the $130\,\mathrm{cm}$–high window, from what height above the top of the window was it dropped? [Wolf 2-78]

I will set up the vertical coordinate y as shown in Fig. 2.13. The origin is at the place where the balloon was dropped, and we don't know how far above the window that is. Note, the y axis points downward here, so that y as a function of time is given by $y = \frac{1}{2}gt^2$.

Using this system, yet the y coordinate of the top of the window bet y_1 and the bottom of the window be y_2. Suppose the balloon crosses the top of the window at t_1 and the bottom of the window at t_2. The problem tells us that

$$y_2 - y_1 = 1.30\,\mathrm{m} \qquad \text{and} \qquad t_2 - t_1 = 0.22\,\mathrm{s}$$

Using the equation of motion for the balloon, we have

$$y_1 = \tfrac{1}{2}gt_1^2 \qquad \text{and} \qquad y_2 = \tfrac{1}{2}gt_2^2$$

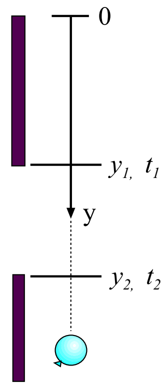

Figure 2.13: Diagram for the falling object in Example 21.

In fact at this point the problem is really solved because we have *four* equations for the *four* unknowns y_1, y_2, t_1 and t_2. We just need to do some math!

One way to solve the equations is to substitute for the y's as:

$$y_2 - y_1 = \tfrac{1}{2}gt_2^2 - \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}g(t_2^2 - t_1^2) = 1.30\,\mathrm{m}$$

But here we can factor the term $t_2^2 - t_1^2$ to give:

$$\tfrac{1}{2}g(t_2^2 - t_1^2) = \tfrac{1}{2}g(t_2 + t_1)(t_2 - t_1) = 1.30\,\mathrm{m}$$

This gives us $t_2 + t_1$:

$$t_2 + t_1 = \frac{2(1.30\,\mathrm{m})}{(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})(t_2 - t_1)} = \frac{2(1.30\,\mathrm{m})}{(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})(0.22\,\mathrm{s})} = 1.206\,\mathrm{s}$$

Adding this to the equation $t_2 - t_1 = 0.22s$ gives

$$2t_2 = 1.43 \qquad \Longrightarrow \qquad t_2 = 0.71\,\mathrm{s} \qquad \Longrightarrow \qquad t_1 = 0.49$$

And then the equation for y_1 gives us

$$y_1 = \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(0.492\,\mathrm{s})^2 = 1.19\,\mathrm{m}$$

so that the balloon began its fall $\boxed{1.19\,\mathrm{m}}$ above the top of the window.