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UK Equity Release Mortgages: a review of the No Negative Equity Guarantee

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UNIVERSITY OF KENT

CEQUFIN

**UK Equity Release Mortgages: a review of the No
Negative Equity Guarantee**

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Executive Summary

The main points coming out of this research study, taking the point of view of the ERM provider, are the following:

- All models are wrong but some are useful as George Box said, so we prefer selecting as the data generating model for house prices a model that can forecast accurately house prices out-of-sample, for short-to medium-term horizon.
- The ARMA-EGARCH family of models are suitable for time-series with serial correlation and volatility clustering such as Nationwide house price series. The ARMA(4,3)-EGARCH(1,1) (described in Section 3.3) outperforms the GBM model under the real-world measure in terms of forecasting short- and medium-term house prices in the UK.
- Changing the method of parameter estimation may give different results for GBM parameter estimation; changing the length of historical time-series also affects parameter estimates.
- The volatility estimates for the same real estate index may vary with the data generating process assumed, with the econometric method used for parameter estimation, the period of estimation. It is also different across different regions in the UK.
- Based on the Nationwide index data, a range of values between 3.85% to 6.5% seems representative for GBM volatility parameter (see Table 2) and 10% or 13% is then more of a stressed scenario value.
- The NNEG values are very sensitive to LTV assumptions so the design of the ERM product is important for risk management purposes. The NNEG valuations are also very sensitive to the roll-up rate R . Since this is a fixed-rate in the UK, an important risk-management control can be obtained at the outset, when the loan is issued.
- The GBM-risk-neutral and the computationally equivalent Black model do not satisfy the theoretical foundations to be used for valuations of NNEGs. GBM is not only *statistically* unsound. Short-selling is currently impossible in housing spot markets. In addition, there is no futures market on residential property in the UK so Black 76 does not apply here.
- Under current market conditions the GBM-risk-neutral/Black 76 may inflate the NNEGs values through higher than necessary volatility at long horizons. This effect may impact on the availability of ERMs and the final cost carried by the borrowers.

- The GBM-risk-neutral/Black 76 model may swing the opposite way when the risk-free rates are larger than the rental yield, overestimating future house prices and underestimating NNEGs.
- For short horizons, such as two years and ignoring seasonality, it is possible for the Black-Scholes model to give similar forecasts with ARMA-GARCH models, in very calm market conditions. For long horizons however, by construction, the variance of house price capital returns increases linearly with time.
- The GBM-risk-neutral/Black 76 is currently inappropriate theoretically, but this does not imply that the risk-neutral valuation approach is inadequate. On the contrary, for the valuation of NNEG we suggest the following: select an appropriate model for house price dynamics, risk-neutralise the process and use Monte Carlo simulations to get house prices in this risk-neutralised world and then value the NNEG.
- It is possible to have almost the same NNEG valuations under both GBM and ARMA-EGARCH approaches, particularly in some specific scenarios such as low risk-free rates or high rental yield and mainly at very high roll-up rates. Hence, in the deep in-the-money NNEG region the GBM-risk-neutral/Black 76 and the ARMA-EGARCH model give almost the same results.
- The GBM-risk-neutral/Black 76 NNEG values are most of the time above the ARMA-EGARCH NNEG values because the NNEG put option gets in the money at longer horizon and GBM has returns variance increasing linearly with time whereas the ARMA-EGARCH is more mean-reverting.
- For the GBM-risk-neutral/Black 76, computationally the NNEG values are driven by the $r - g$ difference, where r is the risk-free rate and g is the rental yield and by the volatility σ .
- A positive rental yield larger than a low risk-free rate will enforce low level projections of house prices under a risk-neutral valuation approach, implying high NNEG values calculated with a risk-neutral measure. At the same time, it is possible to see high levels of projected house prices, under a real-world dynamics, which will imply low NNEG valuations. In relative terms then, the GBM-rw NNEG values will be lower than the GBM-rn NNEG values.
- We derive a rough estimation of rental yield that comes to 1% without taking into account running costs.

- It is difficult to understand the role played by the deferment rate, other than trying to impose some model-free no-arbitrage boundary. Deferment rate cannot be determined reliably at this moment in time.
- A no-arbitrage condition for house forward prices can be determined in terms of buying and selling costs.
- Cash-flow analysis highlights that the main NNEG risk is focalised after 10 years after launch of the portfolio, with 25 years the most critical time.

1 Introduction

The reverse mortgage is a financial instrument that can be tracked back to the 1960s in the United States, with more activity reintroduced during the early¹ 1980s, before spreading to the United Kingdom where it is called equity release mortgage (ERM), in the mid- to late 1980s. It has been reinvigorated worldwide in the aftermath of the subprime crisis, this product being popular in the USA and the Caribbean, in the United Kingdom and some European countries (France), but also in the Far East countries like Japan, Korea, Hong Kong, Singapore, Australia, see AARP (2005), Addae-Dapaah & Leong (1996), Chou et al. (2006), Ma & Deng (2013), Mitchell & Piggott (2004).

Under new regulations, ERMs have been endorsed by Robert Merton as a viable source of funding for the elderly, see Rosato (2016). Moreover, Merton & Lai (2016) discuss a structural design of ERMs that is meant to improve the risk sharing between the borrower and the lender while also examining the role of the regulator in the ERM process.

In the United Kingdom, there is often a guarantee embedded in the ERM contract stipulating that any excess of the accrued loan amount above the sale value of the property after the exit event will be written off by the lender, subject to certain conditions. This is the no-negative-equity-guaranteed (NNEG) condition that is the primary concern with ERMs for lenders. In the United Kingdom, an ERM must incorporate an NNEG in order to meet the Product Standards within the Statement of Principles of the Equity Release Council².

Financing an ERM portfolio poses several challenges because of the long and uncertain maturity profile of the assets. Securitisation used to be a route to get funding and structure the various risks off-balance sheet in a form attractive to medium term note investors. Several such securitisations have been launched in the UK.

An issuer of an ERM has to consider many factors that contribute to the price of the ERM and subsequent valuations. The main factors are age of borrower(s), initial house price, loan-to-value (LTV), house price growth, current risk-free rate, roll-up rate to be applied on the loan, mortality tables, long term care (LTV) incidence, prepayment rates, current yield curve, forward yield curve, funding issues if necessary, idiosyncratic risk due to postcode house price differences, ratings requirements if any, regulatory requirements (Solvency II) and most likely the list is not exhaustive. In some countries, such as Japan, the regulator fixes some important elements in the calculation of NNEG.

In this study, we focus more on the NNEG valuation from a model risk perspective and the associated valuations with various other important cash-flows defining various costs of funds. An excellent description of various ERM type contracts can be found in Hosty et al. (2008). The research has two parts. First, we focus on understanding the issues

¹The Federal Home Loan Bank Board approved ERMs in 1979.

²See www.equityreleasecouncil.com/ship-standards/statement-of-principles

around NNEG valuation and looking in detail at sensitivities (model sensitivity, parameter estimates sensitivity, loan and borrower characteristics sensitivity). The second part of the research (not yet in this version) takes a portfolio view and it considers various costs of funds (funding costs, counterparty credit risk, residual hedging costs, cost of solvency capital, NNEG, other sources of funds).

1.1 Mechanics of ERMs

ERMs must be the primary debt against the house that is used as collateral. The amount to be borrowed under an ERM, sometimes called the principal limit, is determined in a direct relationship to the house value. There are no credit requirements on the borrowers other than keep up with paying their taxes and maintenance costs, paying service charges. The ERM can be seen as a portfolio of an income security and a crossover put option that is automatically applied at termination, effectively posting the house as collateral in the loan back to the lender even if the accumulated outstanding balance is larger.

Depending on jurisdiction, there could be a variety of additional costs related to an ERM. These include upfront costs for setting up the deal, a monthly charge for securing the funding of the loan, monthly servicing fees in case the ERM is not on a lump sum basis. Other important issues related to ERMs are design, securitisation and risk management under Solvency II. Papers covering some of these issues are Andrews & Oberoi (2015), Pfau (2016), Merton & Lai (2016). Rufenacht (2012) is an excellent reference for pricing embedded options in insurance products using a market consistent approach. Nakajima & Telyukova (2017) and Blevins et al. (2017) consider quantitative analyses of ERM borrowers.

1.2 Policy consideration

In his letter to Mark Carney, the Governor of Bank of England, Philip Hammond from HM Treasury, stated:

In discharging its general functions, the PRA must also have regard to the regulatory principles set out in Section 3B of the Act, which are:[...]

the principle that a burden or restriction which is imposed on a person, or on the carrying on of an activity, should be proportionate to the benefits, considered in general terms, which are expected to result from the imposition of that burden or restriction.

The key word here is “proportionate” and this is why it is imperative to allow insurers to conduct internal calculations on the risks associated with ERMs. In their document Prudential Regulation Authority (2017), the PRA made it clear that they will gauge the

allowance made for the NNEG risk against its view of the underlying risks retained by the issuer. Their assessment is spanned by the following four principles,

1. Securitisation where firms hold all tranches do not result in a reduction of risk to the firm.
2. The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG or the present value of deferred possession of the property providing collateral.
3. The present value of deferred possession of property should be less than the value of immediate possession.
4. The compensation for the risks retained by a firm as a result of the NNEG must comprise more than the best estimate cost of the NNEG.

In Prudential Regulation Authority (2018a) there is a substantial section on feedback to responses received on various risk-calculation issues on ERMs. On point 2.29 the PRA considers that the Black-Scholes formula is still appropriate for NNEG put option valuation, but in CP13/18 they also made it clear that other option pricing frameworks may be used as long as it can be demonstrated that valuations meet the four principles enumerated above.

Black-Scholes formula has been reiterated in Prudential Regulation Authority (2018b), that describes the final methodology on managing illiquid unrated assets and equity release mortgages. The formula is described with two fixed values for the main two parameters that are difficult to estimate, the volatility of the house price $\sigma = 13\%$ and the **minimum** deferment rate $q = 1\%$.

1.3 Summary of the components of the proposed valuation process

The valuation process for NNEG has to clear some important hurdles:

- A1** Identify a suitable economic scenario generator including the house price index under the real-world or physical measure P . This can be useful for other risk-management calculations such as value-at-risk or expected-shortfall.
- A2** Identify a suitable mechanism for switching from real-world measure P to risk-neutral measure Q . This step is called *risk-neutralisation* of valuation calculations.
- B** Specify the model for the random maturity determined by multiple decrements of the ERM incorporating mortality, move to long-term care and prepayment.

C Risk-neutral valuation of the contract (ERM or NNEG).

For step A1 insurers can select their preferred ESG (subject to regulatory approval). For step A2, since this is an incomplete market, there is a need but also some flexibility in selecting a method for risk-neutralisation (e.g. conditional Esscher transform) but various methods vary in terms of theoretical and computational complexity. Insurers also have great flexibility over the choice of maturity distribution model including possible future mortality improvements, prepayment and so on.

The NNEG value can be considered at the portfolio level or at the loan level. In this research we focus our analysis on individual loan NNEG calculation. Future research should continue with investigations on NNEG valuation at portfolio level and the degree of capital savings that can be made due to diversification of portfolios and possible NNEG calculation at portfolio level.

2 Literature Review

Earlier models used to price ERM products used static mortality tables. Thus, the trends in mortality rates for some vintages as well as more extreme mortality jumps observed in society were largely ignored. Chen et al. (2010) circumvented this problem by combining a generalised Lee-Carter model with asymmetric jump effects, with an ARMA-GARCH model for a house price index and keeping the interest rate fixed.

Ma & Deng (2013) presented an actuarial based model for pricing the Korean ERM with constant monthly payments and also with graduate monthly payments indexed to the growth rate of consumer prices. They found that any shock to house prices may impact the younger borrowers more severely.

Wang et al. (2014) developed an analytical formula for calculating the *feasible* loan-to-value (LTV) ratio in an adjusted-rate ERM (RM) applied to a lump sum payment. In their model, interest rates are modelled jointly with the adjustable-rate RM, and the housing price follows a jump diffusion process with a stochastic interest rate. Assuming that the loan interest rate is adjusted instantaneously with the short rate given by a CIR model, they show that the LTV ratio is independent of the term structure of interest rates, even when the housing prices follow an exponential Lévy process. They raise concerns about the sustainability of the ERMs at high levels of housing price volatility.

Shao et al. (2015) consider that there are only two main risks that insurers selling ERMs face, real-estate risk and longevity risk. They investigated the joined effect of real-estate price risk and longevity risk on the pricing and risk profile of ERM loans. Their stochastic multi-period model was based on a new hybrid hedonic/repeat-sales pricing model and a stochastic mortality model with cohort trends (the Wills-Sherris model). They concluded that using an aggregate house price index and not considering cohort trends in mortality

may lead to an underestimation of total risk in ERMs.

The literature on ERMs is expanding rapidly, various papers considering different facets of the ERM risk-management process.

2.1 Risk-neutral Approach

Hosty et al. (2008) describes a risk-neutral (called market consistent) approach where a lognormal model is calibrated to the Nationwide House Price Index with a house price volatility taken at 11%, a value obtained by upgrading the 5% p.a. to a higher value (11%) based on the desmoothing procedure described in Booth & Marcato (2004) for *commercial* properties. Although not named clearly, the data-generating process tacitly assumed by Hosty et al. (2008) for NNEG calculus is a GBM process with the drift calculated on a risk-neutral basis as a the difference between the yield on government stock less a rental yield calibrated from the IPD residential property index.

Since that milestone paper, other papers considered various other models, all using risk-neutral pricing as a valuation principle. Here is a list of models (not necessarily complete) that priced the NNEG using the risk-neutral approach: Hosty et al. (2008), Kogure et al. (2014), Alai et al. (2014), Ji et al. (2012), Lee et al. (2012), Wang et al. (2014), Li et al. (2010), Chen et al. (2010).

2.2 Real-world pricing

When a very good econometric model is identified that fits well the house price data so that forecasting is robust, an alternative can be considered based on the physical or real-world measure. The disadvantage of this procedure is that it requires an issuer risk premium specified exogenously or calibrated from a different market.

Hosty et al. (2008) uses the same GBM model, under a real-world measure, with a drift specified on a best estimate approach (mean value). Remark that for GBM model the volatility should be the same under risk-neutral approach and real-world approach. They recognise some of the shortcomings of this model such as the fact that values of house price index in a future period is independent from preceding periods. At the same time they hint that a mean-reversion approach may also be appropriate.

Examples of real-world pricing methodologies are Chinloy & Megbolugbe (1994), Ortiz et al. (2013), Lew & Ma (2012) and Ma & Deng (2013). Many of these papers follow a deterministic approach to house price growth, which can be quite misleading and dangerous.

2.3 Navigating through NNEG Models

Here we offer a concise list of main points related to models applied to NNEG valuation.

The risk-neutralisation of predictive distributions obtained from discrete time econometric models is obtained through several approaches. The main steps are as follows

1. Fit an econometric model to a time series of a house price index; VAR-DCC/GARCH as in Kim & Li (2017), ARMA-EGARCH was fitted by Chen et al. (2010), Li et al. (2010), Kogure et al. (2014), Yang (2011), Lee et al. (2012); a VAR model was fitted by Shao et al. (2015), Alai et al. (2014).
2. Obtain a predictive distribution for the required horizon under the real-world (econometric) measure.
3. Risk-neutralise the predictive distribution. The actual risk-neutralisation step was done in the literature with several methods:
 - the Esscher transform, see Chen et al. (2010), Li et al. (2010), Lee et al. (2012).
 - the Wang transform (Wang et al. 2014). Li (2010) pointed out that for this transform market price of risk is selected based on subjective choices and parameter uncertainty is difficult to gauge while Tunaru (2015) discussed situations when this transform may introduce arbitrage.
 - the (Bayesian) entropy (Kullback-Leibler) approach; used in Kim & Li (2017), Kogure et al. (2014).
 - stochastic discount factor approach as detailed in Alai et al. (2014) and Shao et al. (2015), following Ang & Piazzesi (2003) and Ang et al. (2006).
4. Apply other models for risks such as mortality, long term care and prepayment that are orthogonal to house price risk.
5. Value the target contingent claim.

One should note that other continuous-time models such as the mean-reverting process in Fabozzi et al. (2012) or in Knapcsek & Vaschetti (2007), a jump-diffusion process as in Wang et al. (2014), Lee et al. (2012) or Knapcsek & Vaschetti (2007), and the Levy process also in Wang et al. (2014), are specified under the real-world measure before switching to a risk-neutral measure.

2.4 The geometric Brownian motion model

Studies that used geometric Brownian motion for house prices related to ERM modelling are Hosty et al. (2008), Kau et al. (1992), Huang et al. (2011), Ji (2011), Ji et al. (2012), Pu et al. (2014). Szymanowski (1994) argued that the dynamics of house prices is well represented by a geometric Brownian motion (GBM).

This is in contradiction with the findings of Case & Shiller (1989) and a large body of empirical evidence (Tunaru 2017). Using a geometric Brownian motion for house prices is wrong for several reasons. First of all, the well-documented serial correlation of returns of property prices is not captured. Secondly, the variance for a GBM increases infinitely with the time horizon. Last but not least, a GBM will not be able to produce a property crash since all paths are continuous. A GBM is used to model house prices in the ERM literature *mainly for computational convenience*.

Recent studies accept that house price time-series exhibit serial correlation that invalidates the GBM assumption (Kogure et al. 2014). Li et al. (2010) considered the Nationwide House Price index and they remarked that, for this property index, a) there is a strong positive autocorrelation effect among the log-returns, b) the volatility of the log-returns varies with time, c) a leverage effect is present in the log-return series. All these three properties invalidate the use of the GBM for house prices.

3 Modelling Approaches for NNEGs

3.1 Valuations Considerations for ERMs

While some models lead to closed-form solutions that are easier to implement and monitor, more advanced models lead to Monte Carlo simulations for pricing the NNEG option. Our calculations are organised on a monthly grid and we are going to follow the cash-flows at the end of each month $i \in \{1, \dots, \eta\}$ where η is an acceptable provisional end maturity given by survival to 100 years³. The calculations in general are done for a loan based on a lump-sum.

We shall denote by $Y_t = \ln\left(\frac{H_t}{H_{t-1}}\right)$ the log-return of the house price index at time t .

3.2 Pricing Principles

The important vectors related to NNEG for ERMs are illustrated in Figure 1. There are two different approaches for calculating NNEGs. One is the market consistent approach similar to stock option pricing and based on the risk-neutral pricing, and the other is the classical insurance pricing basis using a real-world pricing measure.

³Actuarial modelling typically assumes a maximum age of 120 rather than 100. NNEG calculations beyond 100 years are very close to zero due to very small discount factors.



Figure 1: The important vectors for an ERM. The vector of values are generic only for illustration purposes. The excess is calculated as the difference between the loan balance and the accrued funding cost.

3.2.1 Modelling Issues

A very important assumption that simplifies calculations of NNEG value is the following:

Assumption 1 *The loan termination is independent of interest rate and house prices.*

Lower interest rates are convenient to borrowers since their outstanding balance will grow at a lower rate. Refinancing may not constitute an incentive due to the transaction costs and crystallisation of payments to be made to the lender. When property prices decline, say through a recession, this motivates borrowers to keep the ERM alive.

3.2.2 Risk-neutral pricing

In the absence of an underlying market, liquid and free of counterparty risk, it is not possible to have a direct risk-neutral approach. In other words, there is no unique martingale pricing measure. Longevity/LTC and prepayment risks are all defining individually an incomplete market. Moreover, the maturity of the NNEG embedded option is stochastic. The NNEG value at the maturity date T is

$$V(T) = \max[K_T - H_T, 0] \quad (1)$$

where K_T is the accumulated balance of the loan and H_T is the house price, at time T .

Under risk-neutral pricing, there is a martingale measure Q such that

$$V(0) = E^Q \left[\exp \left(- \int_0^T r_t dt \right) V(T) \right] \quad (2)$$

where $\{r_t\}_{t \geq 0}$ is a short-rate process.

A common assumption in the NNEG literature is to assume the independence of interest rate processes from other stochastic process underpinning the modelling of NNEG and furthermore, using $r_t \equiv r$ for computational simplicity and taking as given the termination of the loan at T , the value of the ERM loan is given by

$$A(0) = E^Q [e^{-rT} \min\{K_T, H_T\}] \quad (3)$$

This formula can be further decomposed⁴ as

$$A(0) = E^Q [e^{-rT} (H_T - \max(H_T - K_T, 0))] \quad (4)$$

$$= E^Q [e^{-rT} (K_T - \max(K_T - H_T, 0))] \quad (5)$$

$$(6)$$

Formula (5) gives the value of the risky loan since one can decompose

$$A(0) = \underbrace{e^{-rT} E^Q [K_T]}_{\text{present value of accumulate balance}} - \underbrace{e^{-rT} E^Q [\max(K_T - H_T, 0)]}_{\text{value of the NNEG put option}}$$

whereas formula (4) shows that the value of the ERM can be also seen as the value today of future house possession minus a call option on the value of the house with the strike price K_T

$$A(0) = \underbrace{e^{-rT} E^Q [H_T]}_{\text{prepaid forward on house}} - \underbrace{e^{-rT} E^Q [\max(H_T - K_T, 0)]}_{\text{call option on house}}$$

Thus, once the NNEG value embedded in the loan is determined the value of the loan becomes easily computable since the value of the loan repayment is nothing but a zero-coupon bond, and lenders must have robust valuation tools for the latter.

3.2.3 Risk-neutral world GBM pricing

The GBM dynamics, when specified by under the real-world measure, follows the dynamics given by the equation

$$dH_t = \mu H_t dt + \sigma H_t dW_t \quad (7)$$

For simplicity we denote by $K = L_0 e^{RT}$ the exercise price of our NNEG put option at maturity T , where L_0 is the value of the initial loan. Details of the formulae under GBM

⁴I thank Andrew Cairns for pointing out this dual interpretation, similar to credit markets.

are described in Appendix 11.

The volatility under risk-neutral world dynamics and real-world dynamics is the same. The difference between the two types of modelling comes into the drift that is taken as $r - g$ under the former. Here g is the rental yield that is “assumed” to play the same role extra income such as dividends plays for stock.⁵

The Black-Scholes formula behind the NNEG put option is

$$\boxed{Put(H_0, K, T) = Ke^{-rT}\Phi(-d_2) - H_0e^{-gt}\Phi(-d_1)} \quad (8)$$

where $d_1 = \frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (r - g + 0.5\sigma^2)T]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

3.2.4 Real-world GBM pricing

Under this method securities are priced using real-world probabilities derived from the historical information and a risk-neutral discount rate. Thus

$$\boxed{Put(H_0, K, T) = e^{-r^*T} [K\Phi(-d_2) - H_0e^{\mu T}\Phi(-d_1)]} \quad (9)$$

where $d_1 = \frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (\mu + 0.5\sigma^2)T]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

3.2.5 Black 76 Model

Some (Dowd 2018) argued that the “correct” approach is to use the Black (1976) formula for pricing the NNEG. Under this model pricing the NNEG would be done with the formula

$$\boxed{Put = e^{-rT} [K\Phi(-d_2) - F(T)\Phi(-d_1)]} \quad (10)$$

with

$$d_1 = \frac{\ln(F(T)/K) + 0.5\sigma^2T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

where r is the risk-free rate of interest and $F(T)$ is the *forward* house price for year T , that *also* has the formula

$$F(T) = H_0e^{(r-g)T} \quad (11)$$

where g is the house rental yield and H_0 is the current house price.

3.2.6 Limitations of GBM-rn/Black 76 Model

Computationally, Black 76 and the GBM risk-neutral will give the same results (mnemonically we will call this approach the GBM-rn/Black76) so applying Black76 would validate computationally the Black-Scholes model.

⁵We consider rental yield here in order to be able to compare GBM risk neutral as used by some insurers with other approaches. We do not necessarily agree that $g \neq 0$.

Unfortunately, Black (1976) model cannot be applied in the current context for the NNEG market since there is no futures house price contract currently traded in the UK. The introduction of such a futures contract would complete the market and many of the current challenges in valuing house price contingent claims would be easily solved. One of the major impediments in launching the property futures is exactly the development of a simple and flexible modelling approach for this asset class.

In our opinion, the formula (10) simply does not apply for house prices and this is unrelated to the Gaussian distribution assumption behind the GBM model. The forward contract on a house price cannot be calculated as in (11), simply imitating the no-arbitrage formula for a stock paying dividend, where the dividend yield is replaced by the net rental rate. That formula cannot work because currently we cannot shortsell the value of a house. Hence, the no-arbitrage principle does not apply here to lock in the forward price as in the case of corporate stock.

The NNEG put options are not tradeable instruments, not even instruments. They are embedded options in the ERM contract. Hence, the *usual* futures contracts cannot be linked to the NNEG valuations because the interest carry through the marked-to-market and margining to very long maturities may adversely impact the futures valuation. A period by period total-return-swap contract may be more useful and indeed this instrument was more used in over-the-counter trades on real-estate residential indices such as Halifax. Hence, what *may* work is a forward type of contract. To this end we can remark that the Black (1976) provides an option pricing formula using futures values and not forward values. This distinction is also important in the context of stochastic interest rates and when the futures/forward underlying asset price is correlated with the interest rates. The futures and forward prices are identical when the interest rates are constant but even then we cannot circumvent easily the problem explained above, because of the impossibility of shortselling.

Here are the main reservations about the GBM-rn/Black76 methodology.

- GBM as a data generating process for house prices is totally inappropriate because it ignores serial correlation and stickiness of prices, as well as clustered volatility and downward jumps.
- GBM may forecast inflated values of the house price, as demonstrated in Section 3.5.3. This can be very dangerous for real-world valuations, making the NNEG valuations very small because of the overshooting in house prices at longer horizons.
- The assumptions needed to apply the GBM-rn (Black-Scholes) or Black76 (computationally identical) are not satisfied in financial economics terms.
- Furthermore, GBM-rn/Black76 put values depend heavily on the risk-neutral inflation rate of house prices *taken* as $r - g$. Hence, NNEG value calculated based

on either model may be inflated if $r - g$ stays very small (even negative), say low interest rates or higher rental yield, and may be undervalued if $r - g$ is relatively large, say high risk-free rates and/or low, zero or negative, rental yield.

- While we agree with the principle of risk-neutral valuation this should not be confused or assimilated with the acceptance of GBM model as data generating process.
- In our report, we demonstrate that is possible to find models that are more suitable as data generating processes for the house prices.
- A constant rental yield parameter from one year ahead to a long maturity (45 years) may be unrealistic.

3.3 ARMA-EGARCH Model

The ARMA-EGARCH model is answering two problems encountered when modelling house prices. First we have serial correlation. The ARMA part of the model should be able to capture efficiently this effect. Secondly, negative and positive innovations may have different effects on the conditional volatility, allowing financial markets to react asymmetrically to bad and good news, even though in absolute value those innovations may have the same magnitude (Patterson 2000). The model specification should also ensure that the conditional volatility or variance is always positive.

3.3.1 Model specification under real-world measure

This model is based on a submodel for log-returns and a submodel for conditional volatilities. Hence, as in Li et al. (2010), first we specify an ARMA(m,M)

$$Y_t = c + \sum_{i=1}^m \phi_i Y_{t-i} + \sum_{j=1}^M \theta_j \epsilon_{t-j} + \epsilon_t \quad (12)$$

where $\epsilon_t \sim N(0, h_t)$; and then, for the conditional variance h_t the EGARCH(P,Q) model is specified

$$\ln(h_t) = k + \sum_{i=1}^P \alpha_i \ln(h_{t-i}) + \sum_{j=1}^Q \beta_j [|\tilde{\epsilon}_{t-j}| - E|\tilde{\epsilon}_{t-j}|] + \sum_{j=1}^Q \gamma_j \tilde{\epsilon}_{t-j} \quad (13)$$

with $\tilde{\epsilon}_t = \frac{\epsilon_t}{\sqrt{h_t}}$ is the standardized innovation at time t , see Li et al. (2010) for more details. Here t follows the discrete time grid dictated by the data, either monthly or quarterly for Nationwide index.

Our main interest is to use this model and obtain forecasts of volatility to long horizons that can be used for NNEG valuation.

It follows then that, under the real-world measure \mathbb{P}_t

$$\boxed{Y_t | \mathcal{F}_{t-1} \sim N(\mu_t, h_t)} \quad (14)$$

where $\mu_t = c + \sum_{i=1}^m \phi_i Y_{t-i} + \sum_{j=1}^M \theta_j \epsilon_{t-j}$.

The ARMA and EGARCH exact specifications are selected based on goodness-of-fit diagnostic statistics.

3.3.2 Risk-neutralisation of ARMA-EGARCH

Let T be the longest possible maturity for the ERM product; as an example, for a 65 years old if we consider 100 the longest survivor age then $T = 35$ and let \mathbb{P} be the probability measure associated with the information set \mathcal{F}_T . Consider \mathbb{P}_t be the projected measure \mathbb{P} on the smaller information set \mathcal{F}_t . Following Buhlman et al. (1996), Siu et al. (2004) and Li et al. (2010), for a given sequence of constants $\lambda_1, \lambda_2, \dots, \lambda_t, \dots$ the conditional Esscher distribution $\tilde{\mathbb{P}}_t$ is defined computationally through

$$F_{\mathbb{P}_t}^{\sim}(y; \lambda_t | \mathcal{F}_t) = \frac{\int_{-\infty}^y e^{\lambda_t x} dF_{\mathbb{P}_t}(x | \mathcal{F}_t)}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t} | \mathcal{F}_t)} \quad (15)$$

The key to the risk-neutralisation under the conditional Esscher measure is to observe that the moment generating function of Y_t given \mathcal{F}_{t-1} under $\tilde{\mathbb{P}}_t$ is calculated from

$$E_{\tilde{\mathbb{P}}_t}(e^{z Y_t}; \lambda_t | \mathcal{F}_{t-1}) = \frac{E_{\mathbb{P}_t}(e^{(z+\lambda_t) Y_t} | \mathcal{F}_{t-1})}{E_{\mathbb{P}_t}(e^{\lambda_t Y_t} | \mathcal{F}_{t-1})} \quad (16)$$

Because $Y_t | \mathcal{F}_{t-1} \sim N(\mu_t, h_t)$ so then it can be proved that

$$E_{\tilde{\mathbb{P}}_t}(e^{z Y_t}; \lambda_t | \mathcal{F}_{t-1}) = e^{(\mu_t + h_t \lambda_t) z + \frac{1}{2} h_t z^2} \quad (17)$$

The risk-neutral-measure is identified from the local martingale condition by finding those λ_t^q such that

$$E_{\tilde{\mathbb{P}}_t}(e^{Y_t}; \lambda_t^q | \mathcal{F}_{t-1}) = e^{r-g} \quad (18)$$

with r the risk-free rate and g the rental yield⁶. This gives the risk-neutralising constants

$$\lambda_t^q = \frac{r - g - \mu_t - \frac{1}{2}h_t}{h_t} \quad (19)$$

Combining things together gives the sequence of risk-neutral measures \mathbb{Q}_t such that

$$E_{\mathbb{Q}_t}^{\sim}(e^{zY_t}; \lambda_t^q | \mathcal{F}_{t-1}) = e^{(r-g-\frac{1}{2}h_t)z + \frac{1}{2}h_t z^2} \quad (20)$$

which shows that the risk-neutralization effect is to keep the same type of normal distribution but change by translation the parameters. Thus, under \mathbb{Q}_t , we have that

$$\boxed{Y_t | \mathcal{F}_{t-1} \sim N\left(r - g - \frac{1}{2}h_t, h_t\right)} \quad (21)$$

For pricing the NNEG we need to calculate the following risk-neutral expectation, see Li et al. (2010),

$$e^{-r(k+0.5+\delta)} E_{\mathbb{Q}}\left[\left(L_0 e^{R(k+0.5+\delta)} - H_{k+0.5+\delta}\right)^+\right] \quad (22)$$

For simplicity let us denote by $\tau = k + 0.5 + \delta$ which is the known maturity given by the termination of the ERM and $K = L_0 e^{R(k+0.5+\delta)}$ is the accrued balance at τ which is known. Hence the option above is a put option on H_τ . Now, a correct approach will have to take a path-dependent approach and build recursively the chain of conditional volatilities (variances) to the required maturity. For example, for maturity τ , the house price H_τ can be calculated as

$$H_\tau = H_0 \exp\left(\sum_{i=1}^{i=\tau} Y_i\right)$$

using Monte Carlo simulation based on (21) for the risk-neutral measure and based on (14) under the real world measure. We shall refer to this Monte Carlo simulation approach⁷ as the ARMA-EGARCH risk neutral (ARMA-EGARCH-rn) for the former and the ARMA-EGARCH real world (ARMA-EGARCH-rw).

⁶In the absence of market prices for forwards/futures or total return swaps on property, selecting a martingale measure is done with the conditional Esscher transform. An excellent discussion of technical issues involved with using the conditional Esscher transform to identify a martingale measure under the incomplete market setting, in relation to GARCH models, can be found in Siu et al. (2004). For calibration purposes the local martingale condition described here arises by constructing a self-financing portfolio with one unit of the asset S and all rental cum-income invested in the bank account. Under the bank account numeraire, the portfolio discounted is a martingale. In other words, under the martingale measure as identified above, the normalised gains process is a martingale, see Bjork (2009) for an exposition how to deal with dividend income in asset pricing. The rental income should be considered *net of running costs* where possible. Another difficulty with rental income is that when using pounds rental income, for option pricing purposes the present value of *all future* rental income is needed and calculating that looks very difficult, particularly for long horizons.

⁷A similar procedure applies for the ARMA-GARCH family of models.

3.4 GBM for house price index

We first consider the case where underlying property prices follow a geometric Brownian motion which is written as;

$$dH_t = \mu H_t dt + \sigma H_t dW_t, \quad \mu \in \mathbb{R}, \sigma \in \mathbb{R}_+ \quad (23)$$

where μ is the drift rate of house prices, and σ is the volatility of house price. dW_t denotes standard Wiener process increments under physical measure \mathbb{P} .

This model is referred as the Black-Scholes model in the NNEG literature by an abuse of nomenclature since the Black-Scholes model does not apply at this point in time for real-estate markets. The main advantage of this model is computational, since it leads to closed-form solutions for the NNEG put option prices. The risk-neutralised version

$$dH_t = (r - g)H_t dt + \sigma H_t dW_t \quad (24)$$

is often used directly, particularly when the rental yield g is assumed to be given.

3.5 Parameter estimation under real-world measure

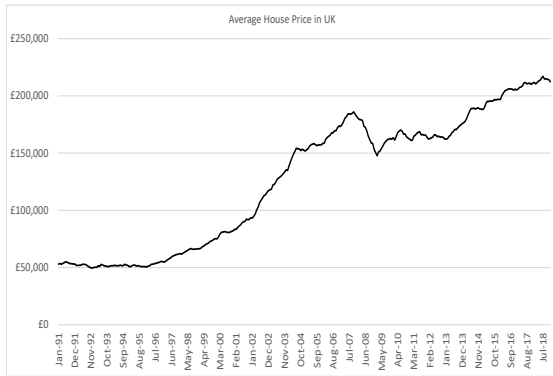
3.5.1 GBM data generating process

For the GBM process specified in (24) we estimate the model parameters on the monthly log-return series of the Nationwide house price series between 1991 and 2016. The reason for using this historical time series rather than the quarterly Nationwide series going back to 1974 as used in other studies was that, for the monthly series, our sample size is almost double the sample size of the quarterly series. Models from the GARCH family require a longer sample in general to be able to fit reliably their parameters. Note though that in Table 2 we analyse volatility estimation under different methods of estimation and over different time periods.

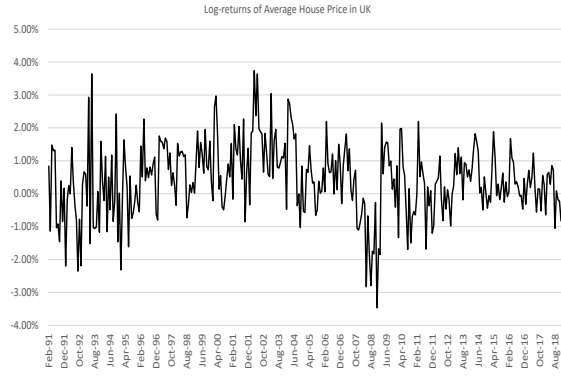
One criticism that can be brought against the use of monthly data is that for Nationwide the historical time series available stops in 1991 and hence is missing the big house price fall between 1988 and 1991, which was larger than the 2007-2010 drop.⁸ Since it is also feasible to use quarterly Nationwide house price series going back to 1952, and we are also employing this quarterly series for robustness checks of our analysis, for forecasting five years out-of-sample.

The Nationwide price series and its corresponding log-returns series used in our analysis are illustrated in Figures 2a and 3d. From these series we keep an out of sample of 24 months 2016-2018 for a comparative forecasting exercise.

⁸I am grateful to Guy Thomas for pointing this out.



(a) levels



(b) monthly log-returns

Figure 2: Nationwide House Price Index Monthly between Jan 1991 and Sep 2018.

For the GBM model, after discretizing at monthly frequency, we use three methods to estimate the parameters from historical data, the maximum likelihood estimation (MLE), method of moments (MM) and generalised method of moments (GMM). In Table 1, we report the estimation results for drift and volatility parameters. For Nationwide we noticed that the MLE and MM methods give very close results. However, the GMM method gives a slightly lower volatility and almost half the drift rate when compared to the other two models. This indicates that the potential of model risk is real.

Table 1: Parameter estimates (annualised) for the GBM process applied to the monthly Nationwide index, between Jan 1991 and Sep 2016, and Halifax Jan 1983 and Dec 2014

Method of Estimation	Nationwide		Halifax	
	μ	σ	μ	σ
Maximum Likelihood (MLE)	5.36%	3.94%	5.80%	3.96%
Generalized Method of Moments (GMM)	3.33%	3.84%	6.45%	2.27%
Method of Moment (MM)	5.36%	3.94%	5.88%	3.96%

The volatilities quoted in other studies are much larger, around 10%. A possible explanation is that they used quarterly data for a longer period going back to 1974 and reporting the sample statistic for standard deviation rather than a volatility parameter associated with a specific data-generating process. Another line of reasoning is to say that the volatility of the residential index is approximately 5%, but then using desmoothing it comes to 11% (Hosty et al. 2008). The volatility of an individual house price may be higher than the index since the index benefits from the diversification effect. Same argument will apply if calculations for volatility are considered at the regional level. At the same time there is a selection bias of properties that become the collateral in the ERM loans that may dampen the volatility effect in the opposite direction.

The exact values used for volatilities may vary from lender to lender based on portfolio composition, vintage, etc.

Table 2: Estimation of annualised drift and volatility parameters from Nationwide quarterly time series 1974-2018 for the entire UK and also across regions, using three methods of estimation: maximum likelihood estimation (MLE), method of moments (MM) and generalised method of moments (GMM).

Region	MLE		MM		GMM	
	μ	σ	μ	σ	μ	σ
Period 1974-2018						
North	6.52%	6.67%	6.52%	6.69%	6.36%	6.51%
YorksHside	6.50%	6.69%	6.50%	6.71%	6.10%	6.49%
NorthWest	6.89%	5.62%	6.89%	5.63%	6.65%	5.53%
EastMids	7.02%	6.03%	7.03%	6.04%	6.83%	5.90%
WestMids	6.93%	6.16%	6.93%	6.17%	6.75%	6.17%
EastAnglia	7.24%	6.92%	7.25%	6.94%	7.01%	6.72%
OuterSEast	7.50%	6.26%	7.50%	6.27%	7.07%	6.07%
OuterMet	7.69%	5.89%	7.70%	5.91%	7.33%	5.75%
London	8.29%	6.42%	8.29%	6.44%	7.91%	6.33%
SouthWest	7.45%	6.18%	7.45%	6.20%	7.14%	6.03%
Wales	6.61%	6.63%	6.61%	6.65%	6.26%	6.52%
Scotland	6.45%	5.43%	6.45%	5.44%	6.25%	5.38%
NIreland	6.71%	8.12%	6.72%	8.15%	6.74%	7.70%
UK	7.07%	5.11%	7.07%	5.12%	6.85%	5.03%
Period covered in Hosty et al. (2008) paper 1974-2006						
North	8.76%	7.02%	8.77%	7.05%	8.67%	6.82%
YorksHside	8.64%	7.01%	8.64%	7.04%	8.80%	6.83%
NorthWest	9.13%	5.64%	9.13%	5.66%	9.01%	5.60%
EastMids	8.91%	6.34%	8.91%	6.36%	8.89%	6.22%
WestMids	8.83%	6.49%	8.83%	6.52%	8.64%	6.48%
EastAnglia	8.95%	7.27%	8.95%	7.30%	9.03%	7.00%
OuterSEast	9.02%	6.47%	9.02%	6.50%	8.93%	6.31%
OuterMet	9.02%	6.00%	9.02%	6.02%	8.92%	5.86%
London	9.38%	6.40%	9.39%	6.43%	9.33%	6.25%
SouthWest	9.27%	6.42%	9.27%	6.44%	9.24%	6.26%
Wales	8.74%	6.62%	8.74%	6.64%	8.54%	6.52%
Scotland	8.44%	5.37%	8.44%	5.39%	8.42%	5.41%
NIreland	9.38%	6.52%	9.38%	6.54%	8.96%	6.59%
UK	8.82%	5.06%	8.82%	5.08%	9.02%	4.95%
Period 2007-2018						
North	0.10%	4.24%	0.01%	4.29%	0.16%	4.25%
YorksHside	0.35%	4.58%	0.35%	4.63%	0.34%	4.52%
NorthWest	0.46%	4.21%	0.46%	4.26%	0.39%	4.21%
EastMids	1.69%	4.08%	1.69%	4.13%	1.76%	3.97%
WestMids	1.63%	4.13%	1.63%	4.17%	1.49%	4.13%
EastAnglia	2.42%	5.21%	2.42%	5.26%	2.65%	4.87%
OuterSEast	3.10%	5.09%	3.10%	5.14%	3.26%	4.78%
OuterMet	3.84%	5.20%	3.84%	5.25%	4.08%	4.96%
London	5.02%	6.24%	5.02%	6.31%	4.29%	6.17%
SouthWest	2.22%	4.64%	2.22%	4.69%	1.92%	4.62%
Wales	0.47%	5.72%	0.47%	5.78%	1.28%	4.97%
Scotland	0.77%	4.60%	0.77%	4.65%	0.60%	4.49%
NIreland	0.10%	10.03%	-1.97%	10.06%	0.25%	9.71%
UK	2.02%	4.42%	2.02%	4.47%	1.96%	4.29%

Here we conducted a sensitivity analysis with respect to the estimation of the parameters

of the geometric Brownian motion when applied to house price time-series in the United Kingdom. Table 2 reports the estimates for rate of growth of house prices μ and volatility σ , under GBM, across various regions, and using three methods of estimation.

There is variability in the estimates of house price expected growth rate and volatility, across regions, depending on the period of estimation and method of estimation. Recall that the estimates for monthly data of the same Nationwide series were slightly different as well. One can note that post subprime crisis the volatility is smaller than before the crisis and the expected growth rate decreased also substantially.

Comparing the volatility value obtained in Table 2 for the period 1974-2006 for the entire UK we notice a small discrepancy between our estimated figures (5.06%, 5.08% and 4.95%) and the reported figure of 5% in Hosty et al. (2008), who also report an annual volatility of 11% after applying a desmoothing⁹ process as described in Booth & Marcato (2004) for **commercial** real estate. Furthermore, Hosty et al. (2008) also argue that the regional effects may also add another 2-3% to the volatility estimate, bringing the volatility estimate to a possible 13% value.

We do not believe that using a volatility estimate derived from a desmoothing process would be valid to use, a) for housing real estate and b) as an input into a Black-Scholes formula requiring a risk-neutral volatility¹⁰. This point is important since in the NNEG literature 10% volatility is taken as indicative for the UK. Based on the results in Table 2 we can see that a value of 10% is already a very conservative stressed upwards estimate. The Nationwide house price index does not incorporate any adjustments for the fact that old houses are being replaced by new and much more expensive houses. There is no clear mechanism on how old houses are replaced by new ones. The longest time series on house prices in the UK goes back to 1952. It is difficult to capture precisely this effect. Insurers can take a conservative view and apply a dilapidation discount as a haircut at the termination of the contract. The dilapidation discount rates should increase with time.

3.5.2 Rental Yield

Under risk-neutral world, the dynamics for both GBM and ARMA-EGARCH models depend locally on the risk-neutral drift $r - g$, where r is the risk-free rate and g is the rental yield. The latter parameter plays the role of dividend yield in equity share price models.

⁹We cannot see any plausible financial economics argument to use desmoothing for a house price index. Smoothing is documented in commercial real estate markets and not in housing markets. A description of this technique, some criticism and some adjustments for improvement are reported in Cho et al. (2003).

¹⁰If a risk manager would like to pursue a smoothing type of modelling for real estate we have described briefly in section 11.4 a procedure that is valid theoretically and that applies also a more rigorous risk-neutralisation procedure.

The dynamics of the GBM as a continuous-time model with drift adjusted for dividend yield is based on the assumption that all dividends are reinvestable immediately in the equity stock. This cannot be the case for the standard house owner because of a lack of granularity (fungibility) of real estate housing markets.

As with the dividend yield, when g increases $r - g$ decreases and therefore the risk-neutral distribution translates to the left. This effect increases the put options. *Ceteris paribus*, one may increase NNEG values by taking higher values for rental yield and decrease NNEG values by decreasing rental yields.

The rental yield data coming out of the Office of National Statistics and presented in Fig. 9 indicates possible challenges in modelling rental yields dynamically.

Remark that the data represents private property rental yields but this pool of properties represents a minority stake in the total pool of properties in the UK. Furthermore, it is not clear whether the properties that will form the collateral in ERM loans are impacted at all by rental yields since lenders will not accept tenancy involved in ERM. Furthermore, while a house price index may be assumed to get some rental income, the ERM borrowers do not have access to this income flow. The rental yield concept may introduce an undesirable asymmetric future valuation view between the borrower and the lender on the same collateral house.

There seems to be a lot of variation in the evolution of rental yields over time, with a large drop observed at the end of 2009 and first half of 2010. There is also great variation across regions in terms of rental yields evolution that needs to be managed idiosyncratically similar to the same issue for volatility. In this study we used an average value of 1% that is representative for 2018 in the UK for baseline scenarios, and we considered higher and lower values (including negative) for sensitivity scenarios discussed later on.

If the concept of rental yields is accepted for housing properties then, given the strong influence on NNEG put options, a dynamic model for rental yields would be desirable but also challenging.

3.5.3 Finding the ARMA-EGARCH model

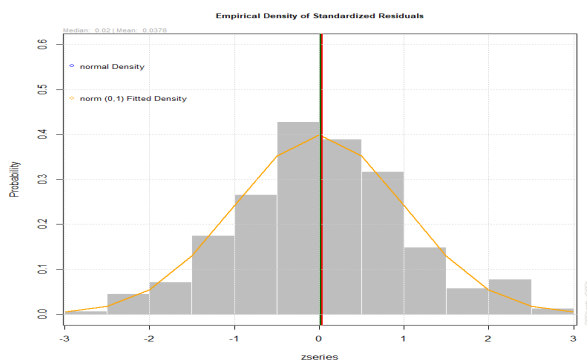
For the ARMA-EGARCH models we consider a forward model selection procedure. From all models that fit well data we select the model using an Occam's razor approach, looking for the simplest possible model (i.e. the smallest number of parameters) that has significant parameters but that also provides a very good fit to the data. The model with superior AIC and BIC goodness-of-fit is preferred.

The model we selected is the ARMA(4,3)-EGARCH(1,1) with parameters in Table 3. There could be other ARMA-GARCH-type models that may provide a better fit than the model we have identified. The universe of ARMA-GARCH-type models is very large

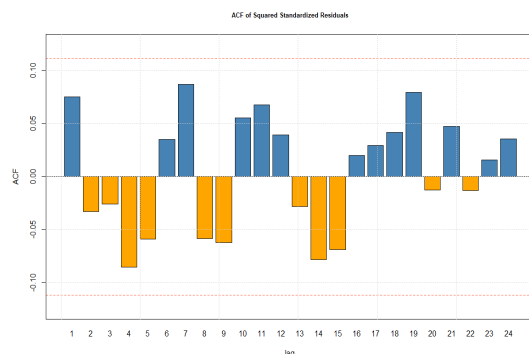
and it is outside the scope of this research to search through the entire universe of this model class. In addition, we worked only with Gaussian errors in the GARCH part of the model but we would highly recommend considering other variants like Student errors.

Table 3: Parameters estimates for the ARMA(4,3)-EGARCH(1,1) model over the monthly Nationwide house price time series between Jan 1991 and Sep 2016.

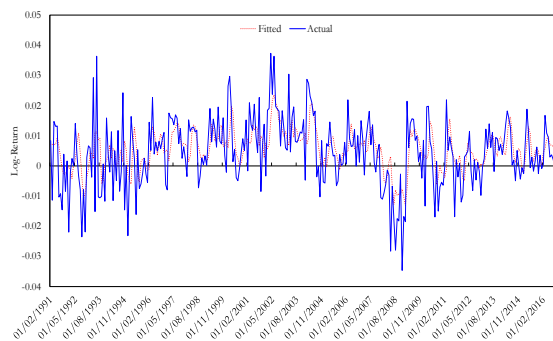
Parameter	Estimate	Std. Error	t-value	$Pr(> t)$
c	0.0071	0.0000	332.5874	0.0000
ϕ_1	1.0550	0.0006	1843.8657	0.0000
ϕ_2	-0.9056	0.0008	-1160.0216	0.0000
ϕ_3	0.1075	0.0003	427.7951	0.0000
ϕ_4	0.3013	0.0006	485.6075	0.0000
θ_1	-0.7604	0.0005	-1562.1327	0.0000
θ_2	1.0739	0.0003	3682.6783	0.0000
θ_3	-0.0465	0.0001	-321.3835	0.0000
k	-0.4436	0.1565	-2.8341	0.0046
α_1	-0.0669	0.0473	-1.4133	0.1576
β_1	0.9529	0.0170	56.1970	0.0000
γ_1	0.1795	0.0829	2.1651	0.0304



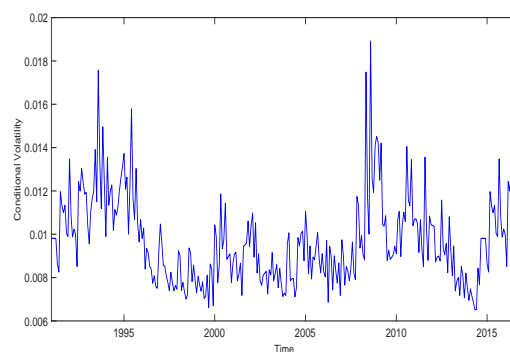
(a) histogram of standardized residuals



(b) autocorrelation of standardized residuals



(c) in sample fitted log-returns



(d) in sample fitted conditional vols

Figure 3: Goodness-of-fit for the ARMA(4,3)-EGARCH(1,1) model for Nationwide House Price Time-series Monthly Jan 1991 to Sep 2016.

The goodness-of-fit usual checks presented in Figure 3 are very good, the in-sample fit is excellent and the conditional volatilities series are in the expected range, varying in

a mean-reverting fashion around the value of 1% (monthly) or 3.16% on an annualised basis. One can also notice conditional volatilities as low as 0.5% (1.6% annualised) or 1.6% (5% annualised).

3.6 Forecasting Comparison

Ultimately, a good model for house price returns should have good forecasting power, at least at short and medium horizon. We retained the out-of-sample period of 2016-2018, monthly, to compare the forecastability of various models.

In Table 4, we report some measures of forecasting accuracy such as root mean squared error (RMSE) and mean average error (MAE) as well as the Diebold-Mariano test (see Appendix 11.5 for more details) for comparing GBM model under different estimation methods with the selected ARMA-EGARCH model, based on the out-of-sample data for monthly Nationwide index. The models in bold provide superior forecasting performance by comparison with the paired model. The test statistic is compared with critical values of standard normal distribution $N(0, 1)$. If we fail to reject the null, i.e. the p-value is between 0.05 and 0.95 at 90% confidence level, then the two models compared produced similar forecasts. Otherwise, the model in the direction of the statistic (1 if negative, 2 if positive) will give better forecasts.

For the short forecasting horizon of two years ARMA(4,3)-EGARCH(1,1) model produces similar forecasts with the GBM specification, under any of the three parameter estimation method.

Table 4: Comparing GBM model under different estimation methods with the selected ARMA-EGARCH model with Diebold Mariano test over the out-sample of 24 months (Oct 2016-Sep 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.00579	0.0004982	
GBM-GMM	0.00581	0.0050080	
GBM-MM	0.00563	0.0041140	
ARMA(4,3)-EGARCH(1,1)	0.0151	0.0126	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-2.3477	0.0278
GBM-MLE	GBM-MM	-2.1684	0.0407
GBM-MLE	ARMA(4,3)-EGARCH(1,1)	0.2327	0.8180
GBM-GMM	GBM-MM	0.4038	0.6900
GBM-GMM	ARMA(4,3)-EGARCH(1,1)	0.2649	0.7934
GBM-MM	ARMA(4,3)-EGARCH(1,1)	0.2637	0.7943

In Figure 4, we illustrate the forecasting error for the out-of-sample Nationwide monthly time series for the last two years. Our results show that it is possible, at a given point in time and for a given forecasting horizon, to have very different models that give similar futures house price values.

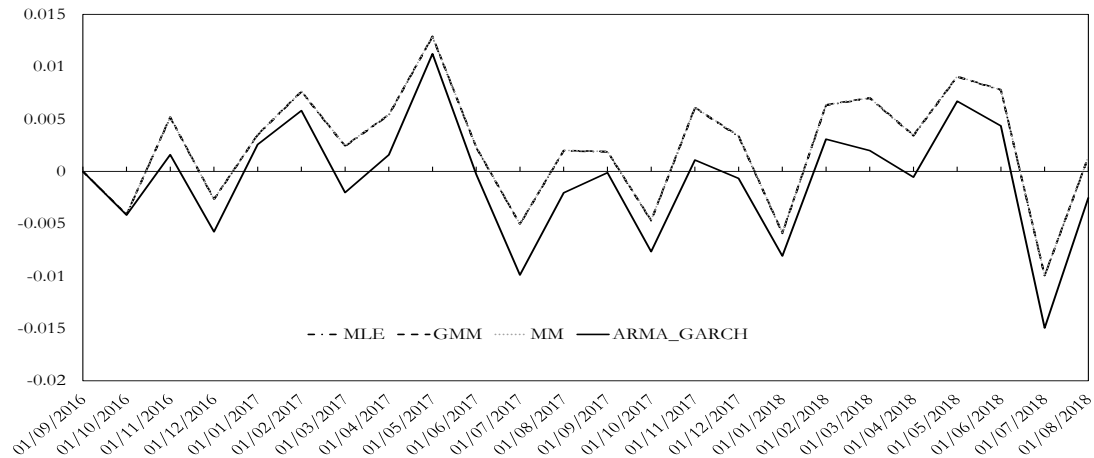


Figure 4: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

One may argue that 2016-2018 was a very benign period for house prices in the UK. We have also redone the analysis for a five year out-of-sample period, with both monthly and quarterly data, the latter going back to 1952.

In Figure 5, we redo the same analysis for the forecasting error for the out-of-sample Nationwide monthly time series with five years out of sample. Now, the ARMA(4,3)-EGARCH(1,1) outperforms the GBM house price forecasting. Moreover, now the MLE estimates for GBM dominates the MLE and GMM method, confirming that there is substantial parameter estimation risk even for such a simple model as GBM.

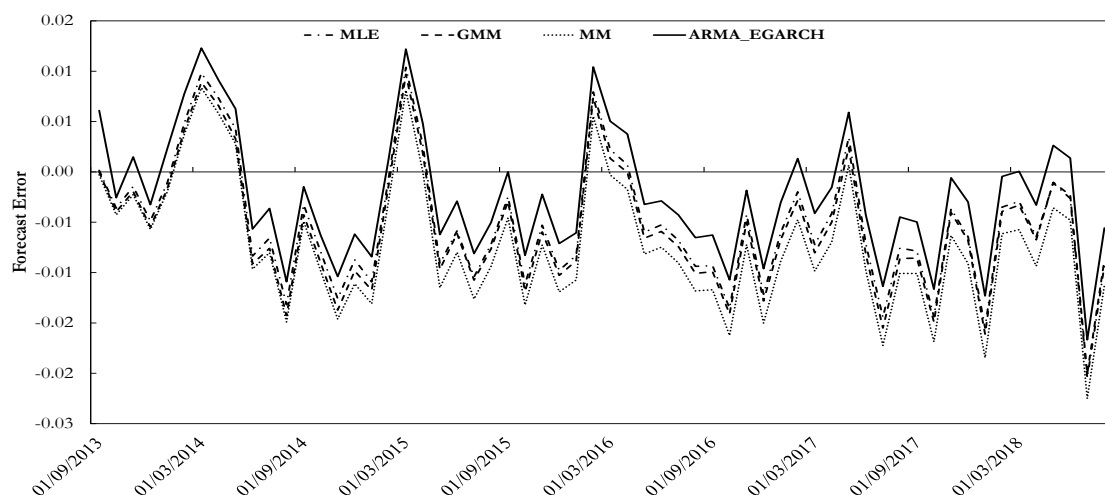


Figure 5: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

Some analysts may like to use the quarterly Nationwide house price data because it goes back to 1952, so it may have more chances to capture more extreme movements. We redid the analysis using this version of Nationwide index time series with five years out of sample analysis. The results are presented in Figure 6 and they show again the superiority of the ARMA-EGARCH model in terms of forecasting future house prices.

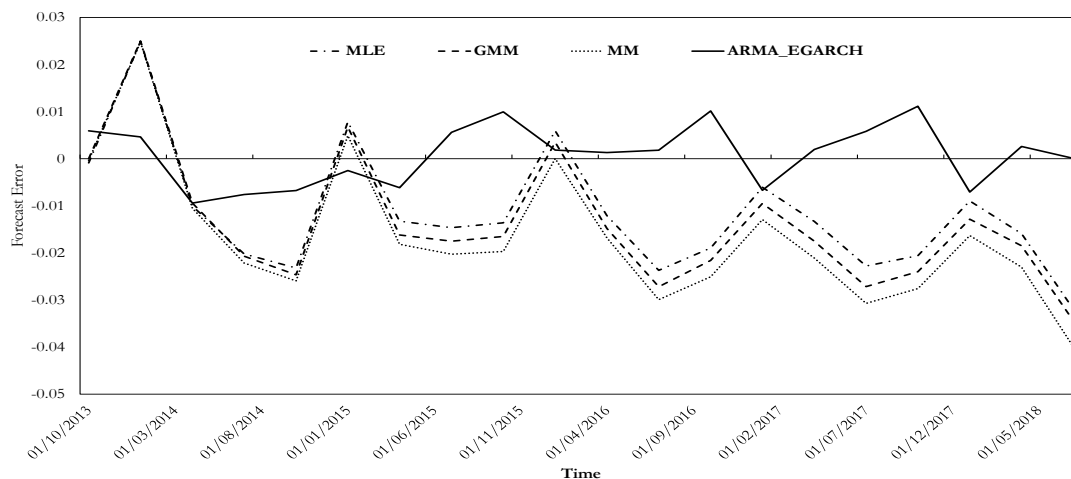


Figure 6: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Quarterly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Q4 2013 to Q3 2018.

In Table 5, we present the forecasting testing result based on monthly frequency and refitted models.¹¹ Even for this much longer period the forecasting under the ARMA-EGARCH model is net superior to the forecasting under GBM. A similar conclusion can be drawn from Table 6 where the same analysis is carried out with quarterly data.

Table 5: Comparing forecasting (monthly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample of 60 months (Oct 2012-Sep 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0079	0.0067	
GBM-GMM	0.0081	0.0069	
GBM-MM	0.0090	0.0078	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0051	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-3.9838	0.0002
GBM-MLE	GBM-MM	-6.7823	0.0000
GBM-MLE	ARMA(4,3)-EGARCH(1,1)	3.7681	0.0004
GBM-GMM	GBM-MM	-6.0371	0.0000
GBM-GMM	ARMA(4,3)-EGARCH(1,1)	3.9739	0.0002
GBM-MM	ARMA(4,3)-EGARCH(1,1)	4.8545	0.0000

¹¹For ease of comparison we retained the GBM model with the three estimation methods and ARMA(4,3)-EGARCH(1,1) that again provides a good fit to the data in-sample.

Table 6: Comparing forecasting (quarterly) under the GBM model with different estimation methods versus the ARMA(4,3)-EGARCH(1,1) model with Diebold Mariano test over the out-sample of 20 quarters (Q4 2012-Q3 2018).

MODEL	RMSE	MAE	
GBM-MLE	0.0147	0.0129	
GBM-GMM	0.0176	0.0158	
GBM-MM	0.0189	0.0170	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0055	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-4.0356	0.0007
GBM-MLE	GBM-MM	-3.9256	0.0009
GBM-MLE	ARMA(4,3)-EGARCH(1,1)	3.6009	0.0019
GBM-GMM	GBM-MM	-3.4400	0.0027
GBM-GMM	ARMA(4,3)-EGARCH(1,1)	4.3231	0.0004
GBM-MM	ARMA(4,3)-EGARCH(1,1)	4.3598	0.0003

For the NNEG put option pricing we are going to simulate forecasting pathways, on a monthly grid, for the conditional variance series $\{h_t\}_{t \geq 0}$. The graphs in Figure 7 describe the conditional simulated pathways for variance series and returns series under ARMA(4,3)-EGARCH(1,1) model. For some months volatilities can spike up leading to potentially high local NNEG values.

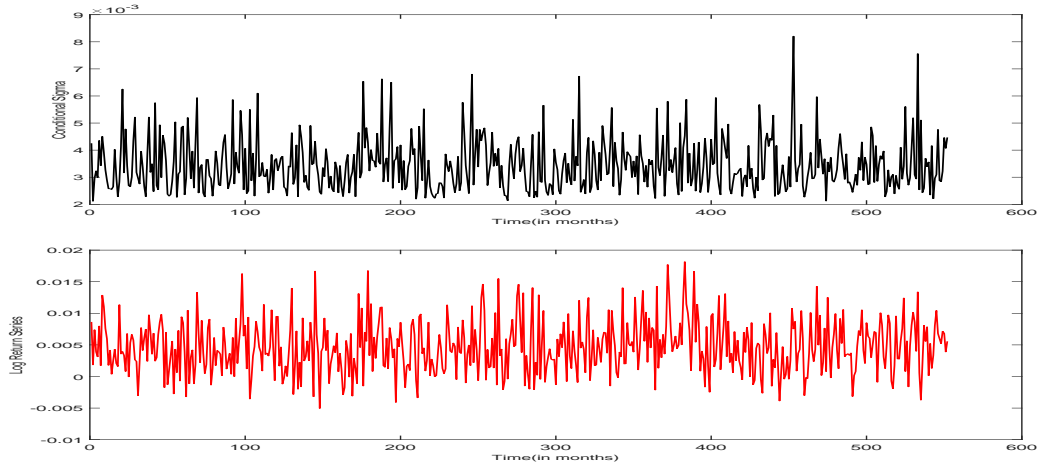


Figure 7: Simulated paths (for illustrative purposes only) for the conditional volatilities and conditional returns under the ARMA(4,3)-EGARCH(1,1) model for 45×12 months ahead at 55 age.

4 Assumptions for Baseline Calculations and Sensitivities

Our baseline scenario inputs (as well as additional scenarios that are part of the sensitivity analysis) are selected based on discussions with experts working on ERMs and using public available tables from Legal & General, Just Group and Equity Release Council,

as of November/December 2018. The assumptions made for the inputs of our analysis reflect current market conditions on the ERM market in the UK.

The baseline scenarios and sensitivity analysis are spanned by the following essential inputs: a vector of LTV loadings for the vector of age group, risk-free rate r or a term structure of risked-free rates $\{r_t\}_{t \geq 0}$; fixed roll-up rate R ; rental yield g , mortality/morbidity/prepayment rates, house price volatility σ .

The role of the assumptions presented here is to contour a profile of standard loans as currently issued on the ERM market in the UK. Furthermore, starting from baseline scenarios described in the market published offers, we can design more realistic what-if scenarios that can be of interested to various market participants.

4.1 LTV

The LTVs are taken from Legal & General Table, see Table 12 in Appendix 10.3, from which we selected only the values for the age groups we focus in our analysis. The values are presented in Table 7. The baseline scenario is the Flexible only option, the other sets of LTVs, Flexible Plus, Flexible Max and Flexible Max Plus are used as sensitivity scenarios.

Table 7: Loan to values (LTVs) for various equity release mortgages issued 29/11/2018. Source: Legal & General

Age	Flexible	Flexible Plus	Flexible Max	Flexible Max Plus
60	17.00%	21.00%	27.50%	29.50%
65	22.50%	26.50%	32.20%	35.10%
70	28.50%	33.00%	36.60%	41.10%
75	32.40%	37.00%	42.00%	47.00%
80	36.50%	42.00%	48.00%	51.50%
85	41.50%	47.00%	50.50%	53.00%
90 and over	41.50%	47.00%	50.50%	53.00%

In addition, as a sensitivity exercise we also consider the following LTV loadings taken from ERC and described in Table 8.

Table 8: Loan to values (LTVs) for various equity release mortgages issued 18/11/2018 and 29/11/2018. Source: ERC

Age of the youngest applicant	ERC1	ERC-Lite	Max ERC	Premier Flexible
60	20.00%	15.00%	25.00%	13.50%
65	25.50%	20.00%	30.00%	19.00%
70	30.00%	25.00%	35.00%	25.00%
75	35.00%	30.00%	40.00%	28.90%
80	40.00%	35.00%	45.00%	33.50%
85	45.00%	40.00%	50.00%	38.00%
90 and over	50.00%	45.00%	55.00%	38.00%

We consider the following average initial house prices in our calculations: for a new lump sum customer 310,000 in line with the Equity Release Council (2018).

The house price risk determines the NNEG risk which is managed through two channels, by charging a portion of the interest rate risk as the customer mortgage rate (called roll-up rate) to cover this potential fall and by insisting on a low LTV. LTVs are in general age-dependent, with lower LTVs for “younger” borrowers and higher LTVs for “older” borrowers, the difference reflecting the expectation of the lender of exit rates. There is also a differentiation by single life and joint life too; in this report we considered only joint life. There are lenders who are willing to give larger amounts of cash to borrowers that can prove that they are in poor health.

4.2 Risk-free rate

For the risk-free rate we take $r = 1.75\%$ that is close to the average 20 year swap rate from Bloomberg in the last three years, see Figure 8. For sensitivity we are going to consider $r \in \{2.0\%, 2.5\%; 1.25\%, 0.75\%\}$. Figure 8 illustrates the evolution of ERM rates in the United Kingdom versus the funding proxy of a 20 year swap rate.

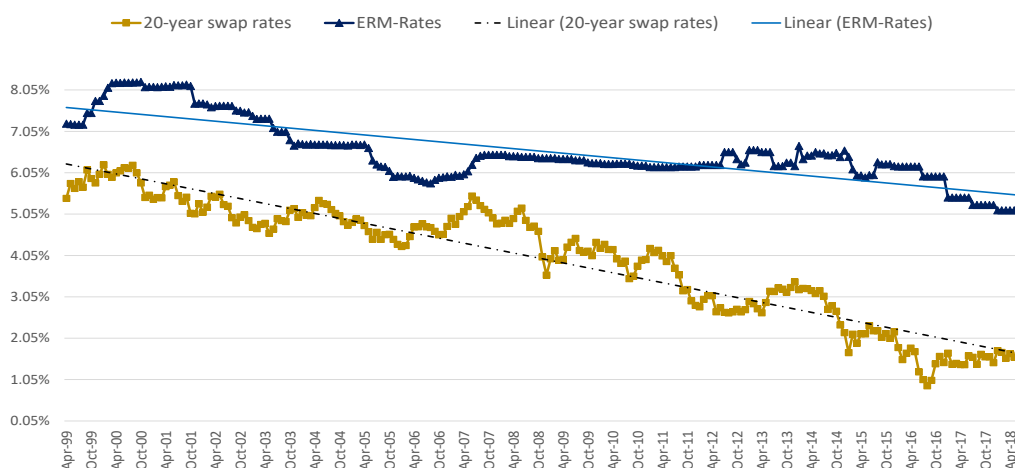


Figure 8: Average of top ERM customer rates against 20 year swap rates: Source Hosty et al. (2008) until 2006, Bloomberg and other combined by us using monthly interpolation.

4.3 Roll-up rate

For the roll-up rate we take the following two baseline rates: $R_1 = 4.15\%$ (AER), that is advertised for properties in London and South East by Legal & General ($R_1 = 4.13\%$ (AER), that is advertised for properties in the Rest of UK by Legal & General) and $R_2 = 5.25\%$ which is in line with the average rates reported in Equity Release Council (2018). For sensitivity we are going to take the baseline scenario rates and increase and decrease them by 1% and by 2% roughly, so the values $R \in \{6.15\%, 7.15\%; 3.5\%, 2.5\%\}$ will describe the sensitivity values of roll-up rates. In addition, in conjunction with the

LTV scenarios Flexible Plus, Flexible Max and Flexible Max Plus, we are going to use $R_{fp} = 4.43\%$ (AER), $R_{fm} = 4.99\%$ (AER) and $R_{fmp} = 5.80\%$, respectively and also for the Premier Flexible LTV curve, $R_{pf} = 4.00\%$ (AER). Furthermore, in conjunction with ERC-Lite and Max ERC LTV scenarios, we are going to use $R_{lite} = 3.85\%$ (AER) and $R_{max} = 4.56\%$, respectively.

4.4 Rental yield

Any additional income produced by an asset, the collateral house in our case, needs to be adjusted for in any contingent claim calculations under risk-neutral (market valuation) approach. The concept of rental yield has been introduced into real estate valuation by analogy with the link between dividends and share prices. However, it can be argued that the buyer of a house is not the equivalent to an investor buying a house as an investment asset. For the majority of buyers, houses play the role of a consumption asset and not that of an investment asset. There is no evidence that rental yields are driving future house prices so the *expected* house prices at various future long horizons cannot be determined with growth models in the same way expected share prices may be determined with growth models linked to dividends.

This issue is also in plain view with other similar assets that are purchased for consumption mainly, such as cars or yachts. Although cars can be rented and yachts can be chartered, the rental rates and the chartered rates are not the main determinants of the prices of cars or yachts.¹² The opposite is true for commercial real estate where indeed, the buyers of commercial real-estate are looking mainly at the rental yield they can generate, and therefore the **commercial** real estate can be considered an investment asset and rental yields do make sense.

Computationally, there are challenges in pinpoint estimation of the rental yield for the UK. The PRA (see PRA CP13-18, paras 2.12-2.15) arrive at 2% by calculating *net rental yield* as the gross rental yield (5%) minus maintenance costs, management costs, voids, with central estimate for net rental yield as 2% but 1% permitted as a *minimum* value. Moreover, it seems that currently some insurers use a value of $g = 2\%$ in line with previous studies such as Hosty et al. (2008), Dowd (2018) and so on (see Appendix 10), while others use $g = 0\%$. Here we are going to take $g = 1\%$ p.a. as our average rental yield representative for 2018. For sensitivity analysis we also take $g \in \{2\%, 3\%; -0.5\%, 0\%\}$. ERM insurers may use external expertise to determine the rental yield appropriate to their portfolios. The more precise calculations are challenging because, for houses the buy-to-let percentage of a houses portfolio is relatively small and it varies geographically with London and South-East as the main area. Hence, the idiosyncratic component of rental yields is quite large. This spatial lack of homogeneity of buy-to-let activity,

¹²We thank Guy Thomas for this last point.

together with the fact that less than 20% of a housing portfolio may be considered to be associated with rented properties, makes it very difficult to consider rental yields as the main drivers of house prices.

Some might challenge “there is no evidence that rental yields are driving future house prices.” Some would argue that the growth in buy-to-let drove increases in prices. The comparison between dividend yield calculation and rental yield calculation refers only to the accounting arithmetic and not to the financial economics attached to it. There are many differences in financial economics terms between dividend payments and rental payments. Dividends may be seen as a drain on company value although many economists may challenge this point of view, rent is not a drain on property value. The discussion may continue with differentiating between investment and consumption assets but some practitioners also argued that commercial real estate is driven by rental value despite rent not being a drain on value. The important point in calculating the rental yield is how this would be paid in real-terms not what it signifies or what it should represent for a house buyer or seller.

4.5 Estimating Rental Yield with Rental Income Data

The split roughly 80% houses owned for consumption and the remaining 20% involved in some way with renting (although the percentage of buy-to-let properties for investment purposes is even smaller) may be used to determine the rental yield if data on the rental income becomes available. The Office for National Statistics has been gathering data on rental yields for a 10% of all properties rented out. From their data we have calculated the monthly sterling rental values average for England taking into account the weights and income given by property type.¹³ The monthly rental yield for England is then calculated by dividing the average sterling rental sum to the average property price in England in that month. In addition, we also calculated *proxy average* quartiles estimates for rental yields using weighted averages of lower, median and upper quartile of monthly sterling rental figures.¹⁴ Figure 9 displays monthly series, average, proxy median and proxy lower and upper quartiles for England. The mean average monthly rental yield over this period is 0.4315% (5.1776% annualised) while the mean proxy upper quartile is 0.48% (5.76% annualised). Note that this rental yield corresponds only to the pool of properties rented out.

¹³We left out the rents coming from room only.

¹⁴The proxy quartiles do not represent actual quartiles since weight averaging the medians will not necessarily produce the median, for example. We produced these proxies to have a rough idea of distribution of rental yields.

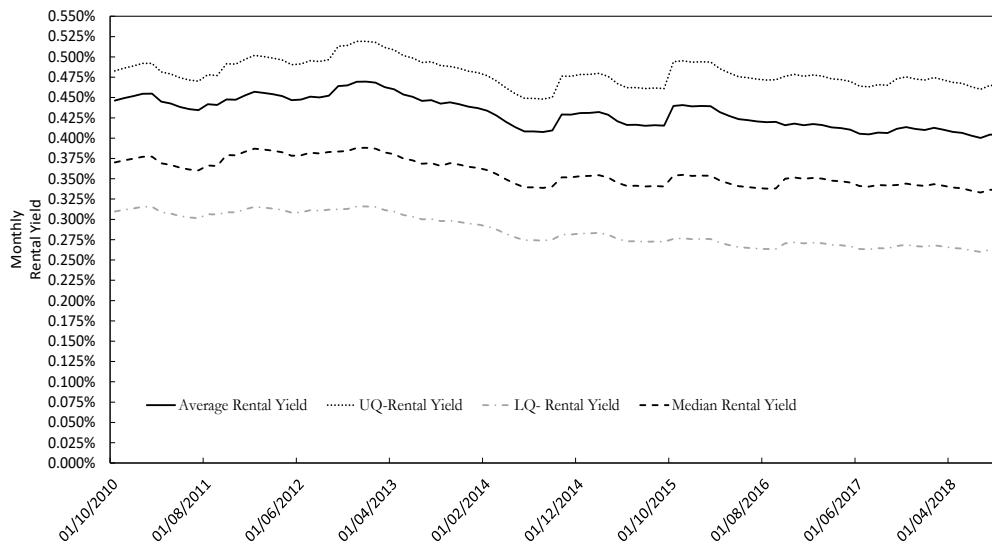


Figure 9: Monthly series, average, proxy median and proxy lower and upper quartiles for England between Oct 2010 and Sep 2018. Source: Author’s calculation based on data from the Office for National Statistics.

According to the Office for National Statistics, there were about 26.4 million households in the UK in the 2012 (following 2011 census) out of which approximately 5 million are rented out properties.¹⁵ Hence less than 20% of properties are rented out. This means that a rough calculation would give a total rental yield, weighted by the 20% representing the actual renting market, of 1.03% ($5.1776\% \times 20\%$) per annum.¹⁶

An even more precise calculation should take into account the *net rental yield* which is calculated as the rental yield net of running costs. The latter is calculated taking into account three elements.¹⁷ First, the voids, defined as the number of months per year the property stays unrented. The usual rule of thumb is to assume one month’s loss of gross rental income per annum, so the sterling pound average rental income will be multiplied by 11/12. Then, letting agent’s fees in the range 10-15% of the rental income plus VAT (12%-18% including VAT) at the current rate of 20%. We can take the mid-value of 15% that needs to be deducted from the resulting sum after applying the voids. The third component refers to maintenance costs that are typically around 15% of the gross rental income, inclusive of any VAT. Hence, agents’ fee and maintenance cost together will erode

¹⁵Personal communication with Rhys Lewis from the Office of National Statistics.

¹⁶This is a controversial debating point and an area of judgement. The 20% weighting is my view as the author and this is open to challenge. It has been debated with other academics and market practitioners who are not entirely convinced about the weighting being applied. Similar to corporate dividend payments though, if a component of the index basket does not make any payment in a given year then its contribution to the extra income yield is marked as zero. In my view this is based on basic accounting rules and it has nothing to do with the financial economics judgements of “what it should be paid or might have been paid”.

¹⁷The values for these elements were selected upon consultation with specialists in the field.

the rental income by 30%. The average net rental yield then following from the above calculations will give an annualised net rental yield of 0.66%.

More realistic calculations carried out by ERM insurers will be aligned with the actual property, including its size/age and location. Some actuaries questioned the $5\% \times 0.2 = 1\%$ because, a) the NNEG put option should be calculated for a single house so in that case the rental yield is either 5% or 0, reflecting the rental status, and b) the observed rental yield is not the same as the potential rental yield.

My answer to those point are as follows. Regarding a) the houses that are collateral in the ERMs, by contractual terms, cannot be rented out since that will complicate legal aspects at termination end. This implies a rental yield of 0. For b) the rental yield was calculated from the rental income that is representative across the properties in the index. If more than 20% of properties become available for renting the rents will dive because of supply and demand. It is not clear what will happen with the house prices, so we cannot say either way what will be the effect overall on the house market.

Recall that the 20% of the houses that produce rental income is not a sample of from the total population of houses that produce rental income. It is the full subset of the population of houses. Hence, the 80% remaining will not have one house that will pay rent. Since we are trying to determine the dynamics of the data-generating process, at the moment, any house price index will have to adjust rental income over the entire population. Likewise, if 80% of the houses will produce that rental income then we would multiply $5\% \times 0.80$ to get the relevant rental yield, and if all houses are rented out producing 5% rental income then 5% is the rental yield on the index.

However, while those issues are important in themselves, our modelling is using a data-generating process for a house price index. We envisage that the NNEG valuations obtained in this way are only “indicative”, say for a house that has exactly the same price as the index. The data-generating process, says GBM, requires the additional income part to be taken into account at the risk-neutralisation stage. Rental yield is needed for GBM and for any other model employing the conditional Esscher transform. Hence, all our calculations are performed based on house index dynamics and not single house calculations.

When conducting loan-by-loan NNEG valuations insurers may rightly adjust the parameters calculated at the index level, such as rental yield or volatility, to reflect the risk in their portfolio. The adjustment may depend on the geographic area, age of the house, proximity to new developments such as airports, or to large employers. Thus, while the index rental yield may be estimated at 1% or 2%, the rental yield used for the individual loans may go up to 3% or 4%, say, and volatility may be 5% for the index, but it may be adjusted to 7% or even 10%, to reflect the basis risk.

Another feasible solution would be to calculate the NNEG values conditioned on the type of the property. Hence, if $\text{NNEG}(1)$ represents the NNEG valuation calculated with

$g = 0$ and $NNEG(2)$ represents the NNEG valuation (under same model) calculated with $g = 5\%$ then by conditioning we would have that

$$NNEG = 0.8 \times NNEG(1) + 0.2 \times NNEG(2). \quad (25)$$

This value is evidently different from the value of the NNEG calculated with a weighted rental yield average.

4.6 Decrement Tables

The mortality table used is the 2015-2017 Office of National Statistics Mortality Table for Male and Female, see Table 11 in the appendix. For the Joint mortality/survival tables we do internal calculations based on suggestions of documents from Knapcsek & Vaschetti (2007).

Our calculations are based on the standard current (period) national mortality tables, rather than on tables with lower mortality (given the socio-economic group involved) with mortality improvements. The latter will produce longer decrement maturities and *ceteris paribus* larger NNEG values. On the other hand, special products offered to particular categories of borrowers have special LTVs and we considered those products in our sensitivity analysis.

For morbidity or long term care (LTC), following discussions with insurance industry experts, we upgraded the adjustment loadings table previously reported in Hosty et al. (2008), to the values in Table 9.

Table 9: Percentage loading to base mortality due to long-term care entry.

Age	Males (%)	Females (%)
≤ 70	6	9
$(70,80]$	8	24
$(80,90]$	10	26
$(90,100]$	8	16

For the prepayment rates we take the following values in Table 10. The rates in the first column were derived by adjusting the initial prepayment rates described in Hosty et al. (2008) to reflect current market prepayment rates based on discussions with three insurers. The other two are stressed scenarios taking into account the insurers view that after 20 years prepayment rates flatten out.

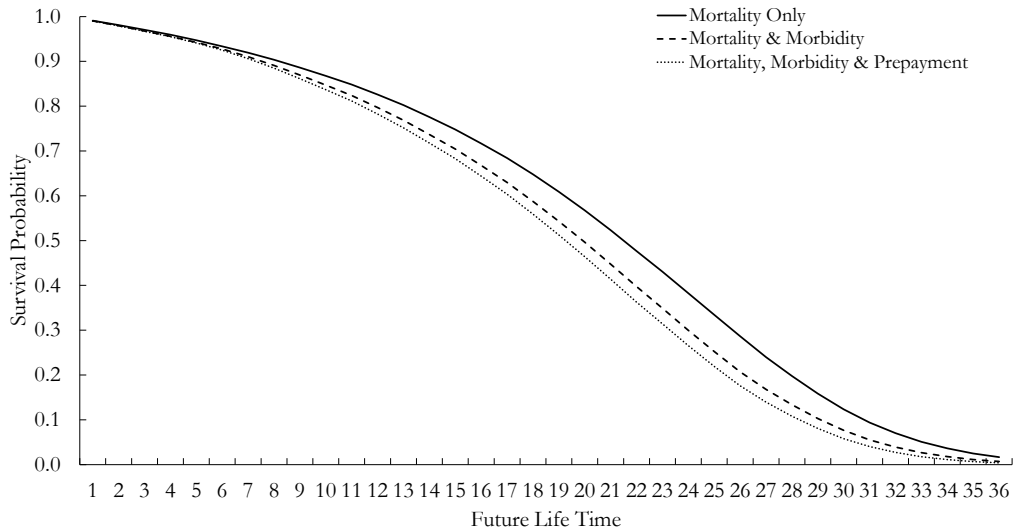


Figure 10: Survival curves under different decrements assumptions based on mortality, morbidity and voluntary prepayment.

Table 10: Prepayment rates

Year	Prepayment rate (%)	First stressed scenario (%)	Second stressed scenario
1-2	1.0	2.5	3.0
3	2.0	3.5	4.0
4-5	2.5	3.0	3.5
6-8	2.0	2.5	3.0
9-10	2.0	2.0	2.5
11-20	2.0	2.0	2.5
21+	2.0	2.0	2.0

The survival curves obtained under various decrements look like those illustrated in Figure 10.

5 Main Results

5.1 Baseline Scenarios

In the absence of market benchmark prices, our sensitivity calculations are gyrating around pre-defined base case scenarios. To this end, we use the following fixed baseline case values for the main drivers of the NNEG value. All calculations reported in this section are done under multiple decrements (mortality, morbidity and prepayment). Like for like calculations for single decrements are available upon request from the authors.

Figure 29 shows the comparative NNEG valuations, henceforth as percentage of lump sum advanced, for the first and second baseline scenario ($r = 1.75\%$, $R = 4.15\%$, $g = 1\%$, $\sigma = 3.90\%$ and standard Flexible LTV vector). For the first baseline, both models

GBM-rn and the Arma-Egarch model give very low NNEG values. The reason for that is a positive risk-neutral drift ($r - g = 0.75\%$) and a relatively small volatility level which indicates that overall the pathways values for house prices will be rising enough to overcome the loan outstanding balance at the rate of $R = 4.15\%$. For the second baseline scenario, the NNEG values for the Arma-Egarch model are about half the NNEG values produced by GBM-rn, and both sets of values are larger than the corresponding values in the first baseline scenario. The main reason for this is the more aggressive roll-up rate $R_2 = 5.25\%$.

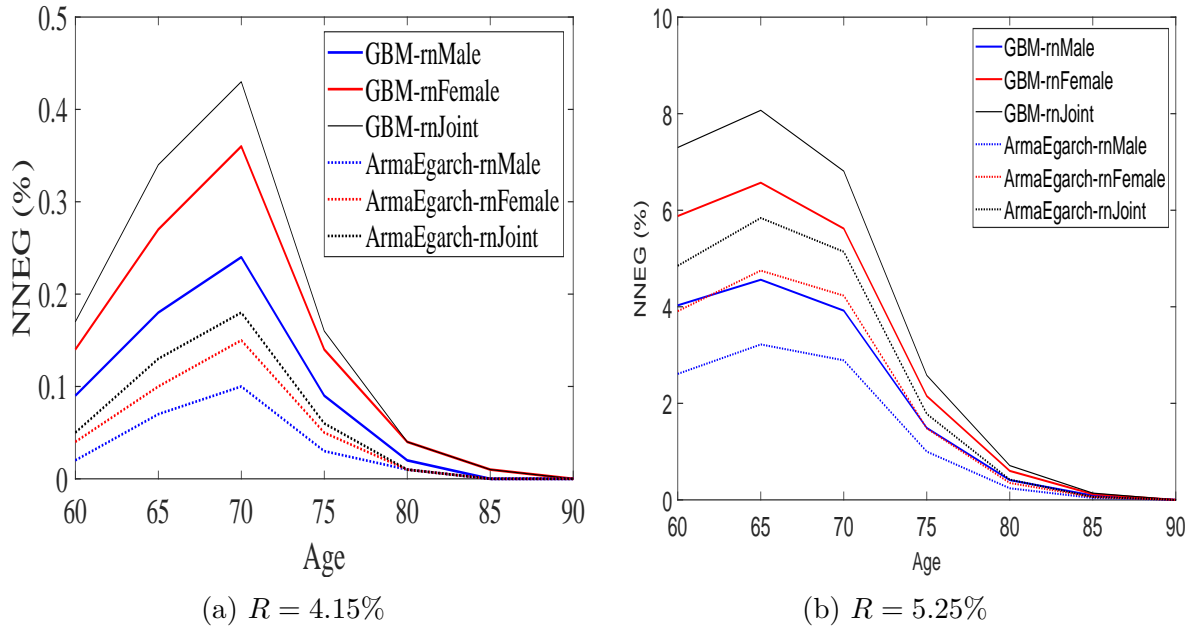


Figure 11: NNEG valuations as percentage of lump sum for GBM-rn and Arma-Egarch-rn , under multiple decrement rates for the two baseline scenario with $r = 1.75\%$, $g = 1\%$, $\sigma = 3.90\%$ and standard Flexible LTV vector valuations.

The NNEG values are higher at earlier ages such as 65, 70 or 75, depending on the market inputs, contractual LTV and other parameters used for valuation, and they decrease rapidly after the age of 75.

We can see that, under current market calibration, the better forecasting model ARMA(4,3)-EGARCH(1,1) has lower NNEGs than GBM calibrated NNEGs. Moreover, a roughly 1% increase in the roll-up rate pushes up the NNEG values two orders of magnitude. The same change in roll-up rate may also determine a switch in the peak of the NNEG from 70 to 65.

This relative ordering is the opposite to what was calculated by Li et al. (2010), due primarily to the assumptions used. Most notably, Li et al. (2010) used a risk free rate of $r = 4.56\%$, a roll up-rate of $R = 6.39\%$ and a rental yield $g = 2\%$. In their GBM calculations, the assumed volatility is estimated over a sample period that is different from what is used in this study. The LTV they used is also very different, since the loan

amount in their study is fixed at GBP 30,000.

To clarify, in our discussion of potential approaches and use of models, we never claim that there is an expected ordering between NNEG calculations under GBMrn and under ARMA-EGARCHrn. Taking GBMrn as the fixed reference calculations, two different models from the same class ARMA-EGARCH may give NNEG results on the other side of the GBMrn. This is possible. Furthermore, the same model refitted over different periods of time may again switch sides versus the GBMrn NNEG values. Finally, there could be several models, all fitting reasonably well the house price time series, which produce NNEG values varying around GBM NNEG values. It would be wrong to select a model just because it gives NNEG values above GBMrn values, or below GBMrn values. Ultimately, we do not have current benchmark values on ERM to calibrate against.

5.2 Comparison of Baseline Scenarios across LTV loadings

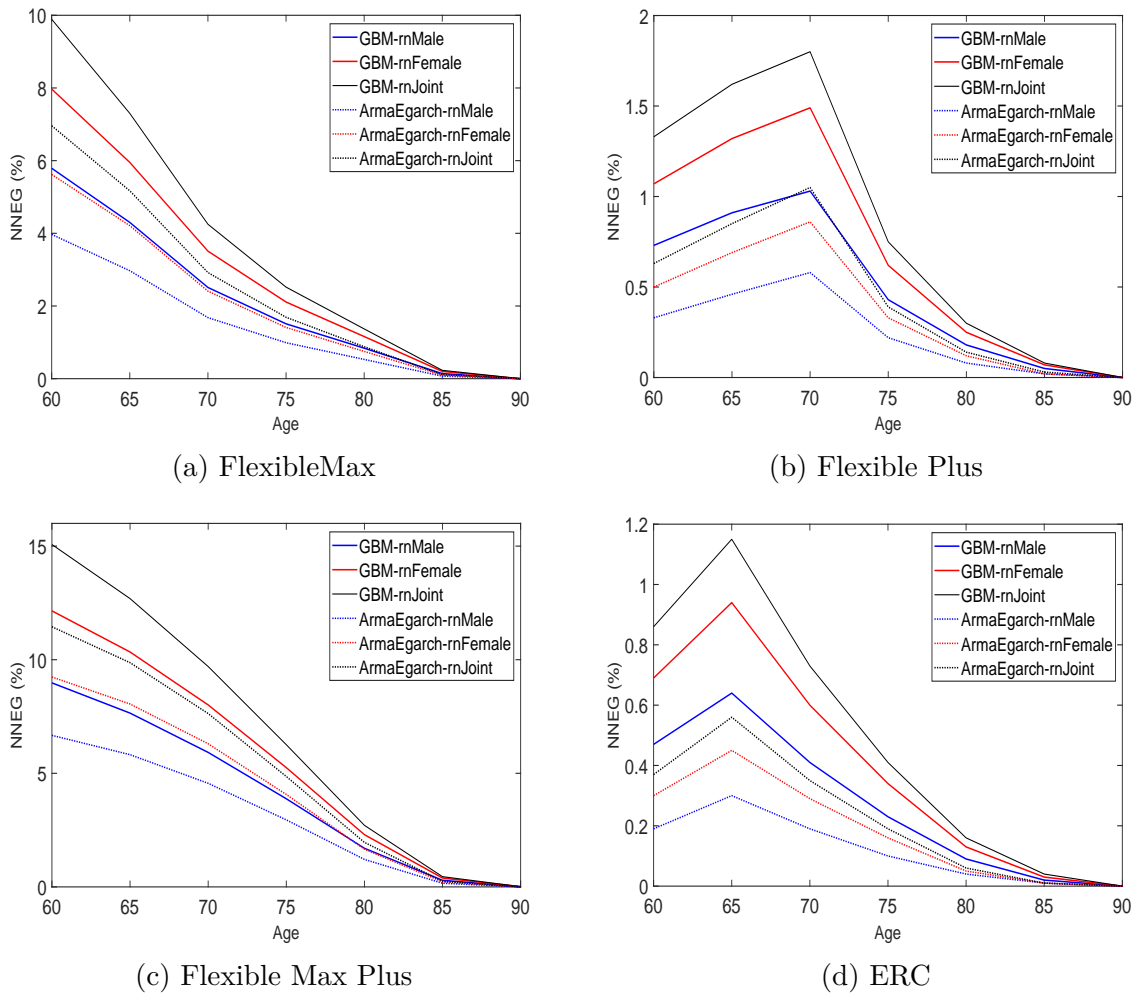


Figure 12: Sensitivity Analysis of NNEG valuation for baseline scenarios w.r.t. different LTV loadings and $r = 1.75\%$, $R = 4.15\%$, $g = 1\%$, $\sigma = 3.90\%$.

The LTVs play a fundamental role in NNEG valuations and they can change the profile of the NNEG vectors as well as their overall magnitude level. Figure 12 shows the comparison of the baseline scenarios under various market LTV loadings. The LTV has a great influence on the final of the NNEG and one can argue that one of the most efficient methods to manage the NNEG risk is to consider carefully the LTV.

The comparative NNEG calculations presented in Figure 12 indicate that higher LTVs lead to a steepening of the NNEG values.

5.3 NNEG calculations under a risk-free curve

Figure 13 shows the NNEG values for the two baseline scenarios under a full risk-free curve described in Appendix 10.10 for 26 December 2018. The calculations indicate lower NNEG values under a full risk-free curve. Notice that ARMA-EGARCH risk-neutral values are re-simulated with adjusted drifts every month based on changing risk-free rates sourced from the full term structure of risk-free rates. Overall, we can see that there are very small marginal changes in the levels of NNEG, but the profiles of NNEG vectors under the two baseline scenarios remain the same. The reason for that is that the only sizeable difference in risk-free rates between our constant rate of $r = 1.75\%$ and the rates indicated in Table 20 is at the front end of the curve. However, the NNEG values at the immediate maturities are zero due to LTV protection.

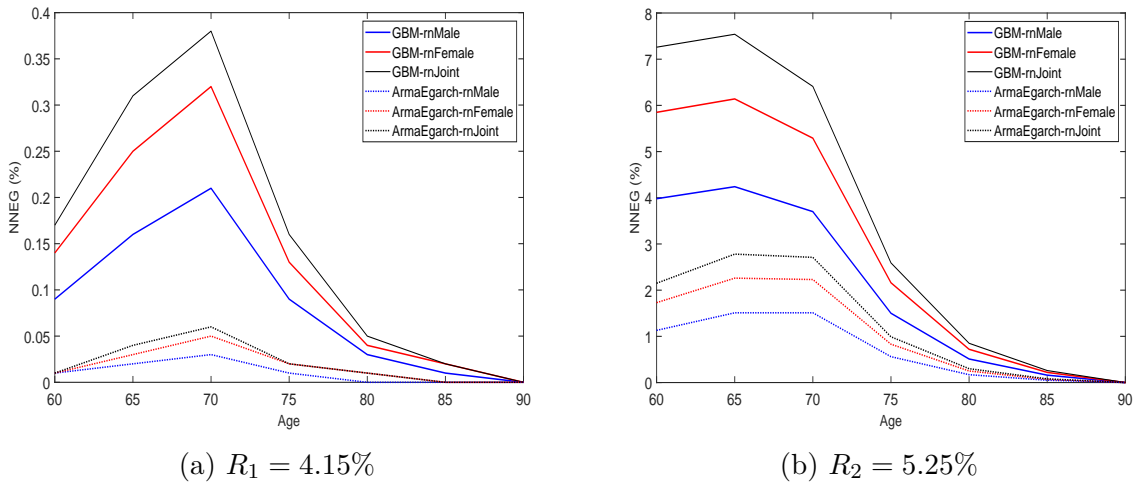


Figure 13: Calculations of NNEGs for baseline scenarios w.r.t. risk-free curve and the two baseline scenarios $R_1 = 4.15\%$, $R_2 = 5.25\%$, $g = 1\%$, $\sigma = 3.90\%$.

Without too much reduction of generality, henceforth we will explore sensitivities to other inputs using a constant risk-free rate(s).

5.4 NNEG sensitivities with respect to main inputs

5.4.1 NNEG sensitivities with respect to risk-free rate

The risk-free rate impacts NNEG calculations in two ways. The obvious way is via discount factors, a lower risk-free rate keeping the back end NNEG put payoffs still high, or, to put it the other way, a high risk-free rate dampening the back end NNEG values. The second channel of interference is the calibration of the conditional Esscher martingale measure using $r - g$ as the local drift, which also appears in the GBM-rn. This risk-neutral local drift makes NNEG put values move in the opposite direction to risk-free rates, the larger the risk-free value the lower the NNEG, and the lower the risk-free rate r the higher the NNEG put value.

The analysis presented in Figure 15 shows that both GBM-rn and ARMA-EGARCH-rn NNEG values decrease when r increases and the NNEGs increase when r decreases. This is intuitively correct, *ceteris paribus* a larger r will shift the projected future house price values in a risk-neutral world upwards, while a lower r will project lower house prices. Furthermore, under Flexible Max Plus LTV loading, when $r = 1.25\%$ or 0.75% , the two valuation approaches give almost identical values. Around current levels of risk-free rate, the ARMA-EGARCH-rn valuations are substantially lower than the GBM-rn valuations. More results under various other LTV loadings are presented in Appendix 12.

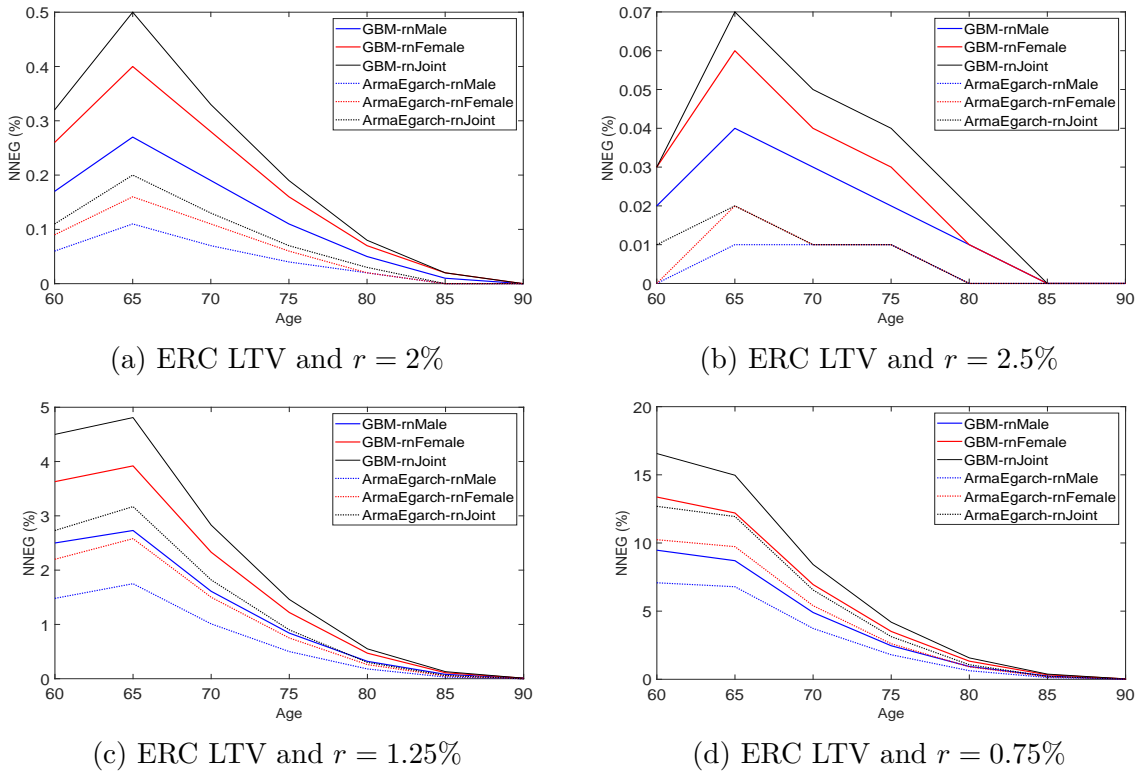


Figure 14: Sensitivity Analysis of NNEG valuation w.r.t. r under ERC LTV and loading and $R = 4.15\%$, $g = 1\%$, $\sigma = 3.90\%$.

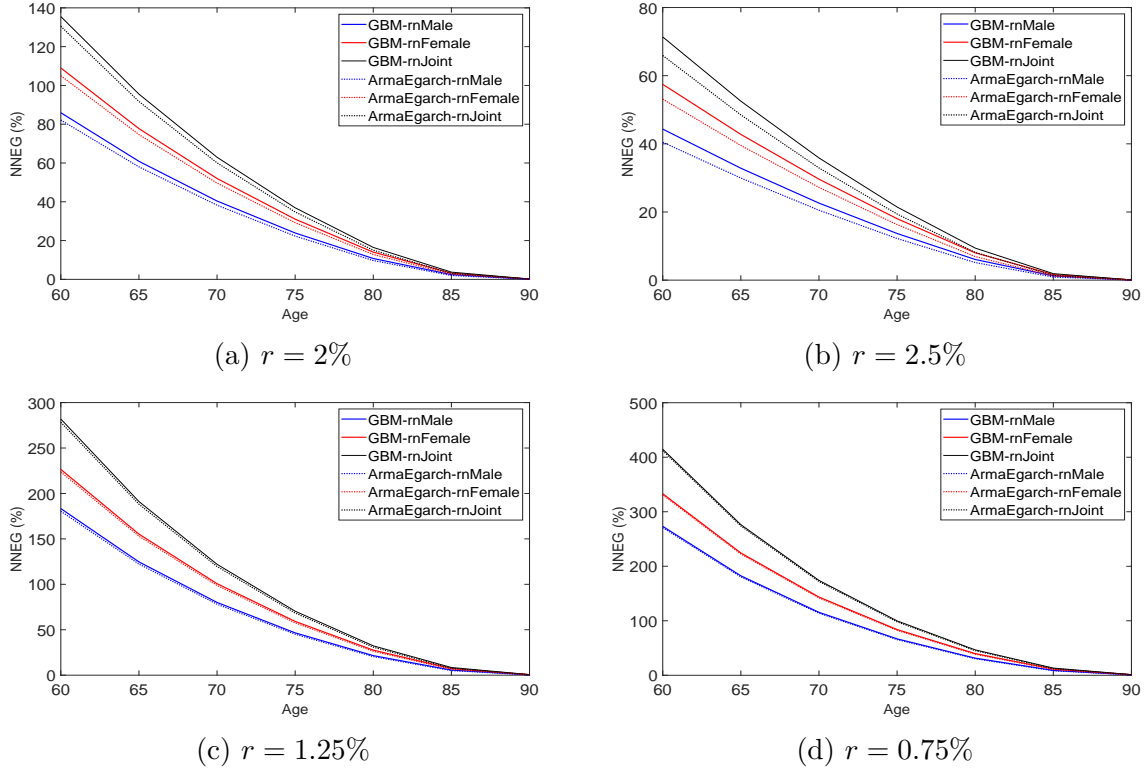


Figure 15: Sensitivity Analysis of NNEG valuation w.r.t. r under Flexible Max Plus LTV loading and $R_{fmp} = 5.8\%$, $g = 1\%$, $\sigma = 3.90\%$

5.4.2 NNEG sensitivities with respect to roll-up rate

The roll-up rate is decisive in the NNEG ending up in the money. It is evident that even a slight increase in the roll-up rate, compounded monthly to 45-55 years, may inflate the loan accumulated balance to very high values. Hence, another good tool for risk-managing the NNEG levels attached to ERMs is having roll-up rates as low as possible.

Figure 16 displays the sensitivities of the NNEG calculations with respect to variations in the roll-up rate. The NNEG values increase dramatically with the increase in the roll-up rate and decrease rapidly with the decrease in the roll-up rate to insignificant values as soon as $R = 3.5\%$. For large LTV levels and large R the valuations between GBM-rn and ARMA-EGARCH-rn approaches become indistinguishable.¹⁸ Note that for Max ERC LTV with $R = 2.5\%$ the NNEG values are virtually zero. More results under various other LTV loadings are presented in Appendix 12.

¹⁸A comparison of NNEG values on the log-scale is presented in Appendix 12.6.

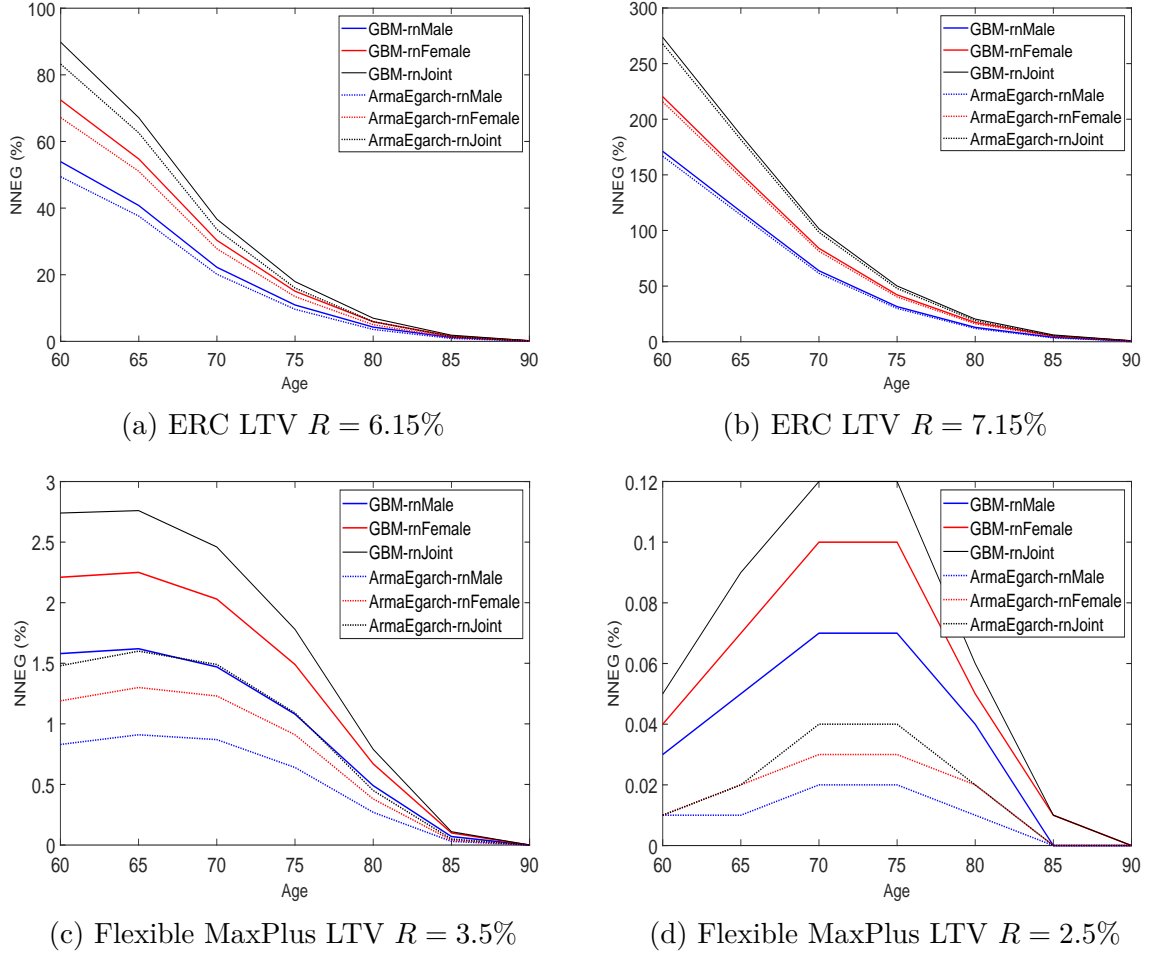


Figure 16: Sensitivity Analysis of NNEG valuation w.r.t. R under ERC and Flexible MaxPlus LTV loading and $r = 1.75\%$, $g = 1\%$, $\sigma = 3.90\%$

5.4.3 NNEG sensitivities with respect to rental yield rate

The rental yield g is another important lever for influencing NNEG calculations. In the models we investigate in this study, the rental yield plays the opposite role to r , such that a high g will computationally give a lower risk-neutral drift (even negative) so the pathway of house prices is trending down, boosting the NNEG put values. The opposite is true for low or even negative g .

The sensitivity of NNEG valuations with respect to the rental yield g under various scenarios are presented in Figures 17. The NNEG values increase with an increase in rental yield and decrease with the decrease in rental yield, for both the GBM-rn and ARMA-EGARCH-rn. Intuitively this is correct, since a larger value for g implies a low or even negative drift in the risk-neutral world so the projected house prices will be lower in the future, implying a higher NNEG value. The opposite works in reverse, a smaller value for g or even zero, as some insurers are using, leads to a more positive drift in the GBM-rn model that will give increased house prices in the future and hence lower NNEGs.

More results under various other LTV loadings are presented in Appendix 12.

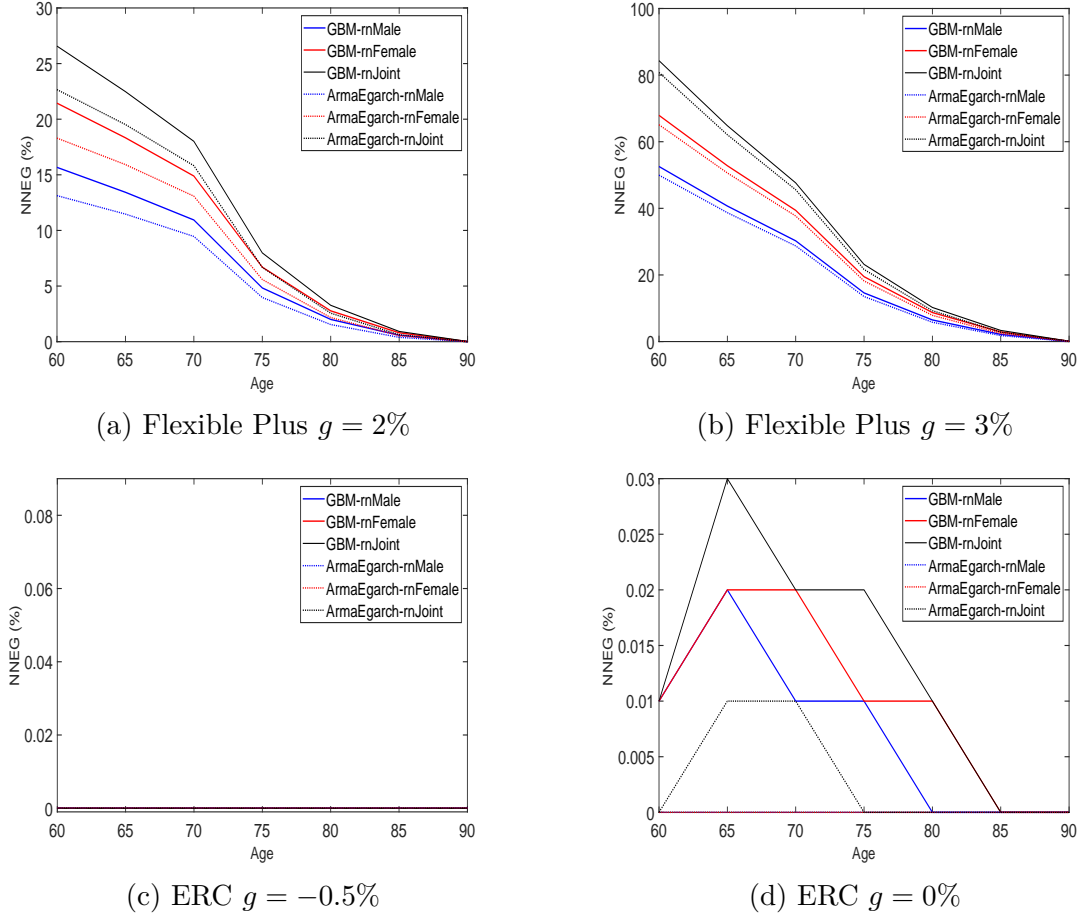


Figure 17: Sensitivity Analysis of NNEG valuation w.r.t. g under ERC with $R = 4.15\%$, and Flexible Plus LTV loading with $R_{fp} = 4.43\%$, and $r = 1.75\%$, $R_{fp} = 4.43\%$, $\sigma = 3.90\%$.

5.5 NNEG sensitivities with respect to house price volatility

Figure 18 illustrates the NNEG valuations with respect to changes in the volatility of house prices that impact the GBM-rn approach.

The volatility of the data generating process employed for house prices plays a key role in any option-type valuation. It is known from option theory that higher volatility will imply higher values for the NNEG put.

For the ARMA-EGARCH volatility parameters as estimated on the Nationwide monthly historical time series, we used an almost identical multiplication factor as coming out from the ratio of GBM volatility in the stressed scenario versus the baseline scenario. For example, when we stressed $\sigma_{GBM} = 10\%$, the ratio to the baseline volatility of $\sigma_{GBM} = 3.9\%$ is about 2.5. Hence, we multiply the series of ARMA-EGARCH volatilities by a factor of 2.5 when redoing the NNEG calculations for the ARMA-EGARCH-rn. Likewise, for other sensitivity values of σ , we multiply the entire vector of ARMA-EGARCH volatili-

ties with the appropriate multiplication factor to preserve the same ratio taken for GBM volatility values.

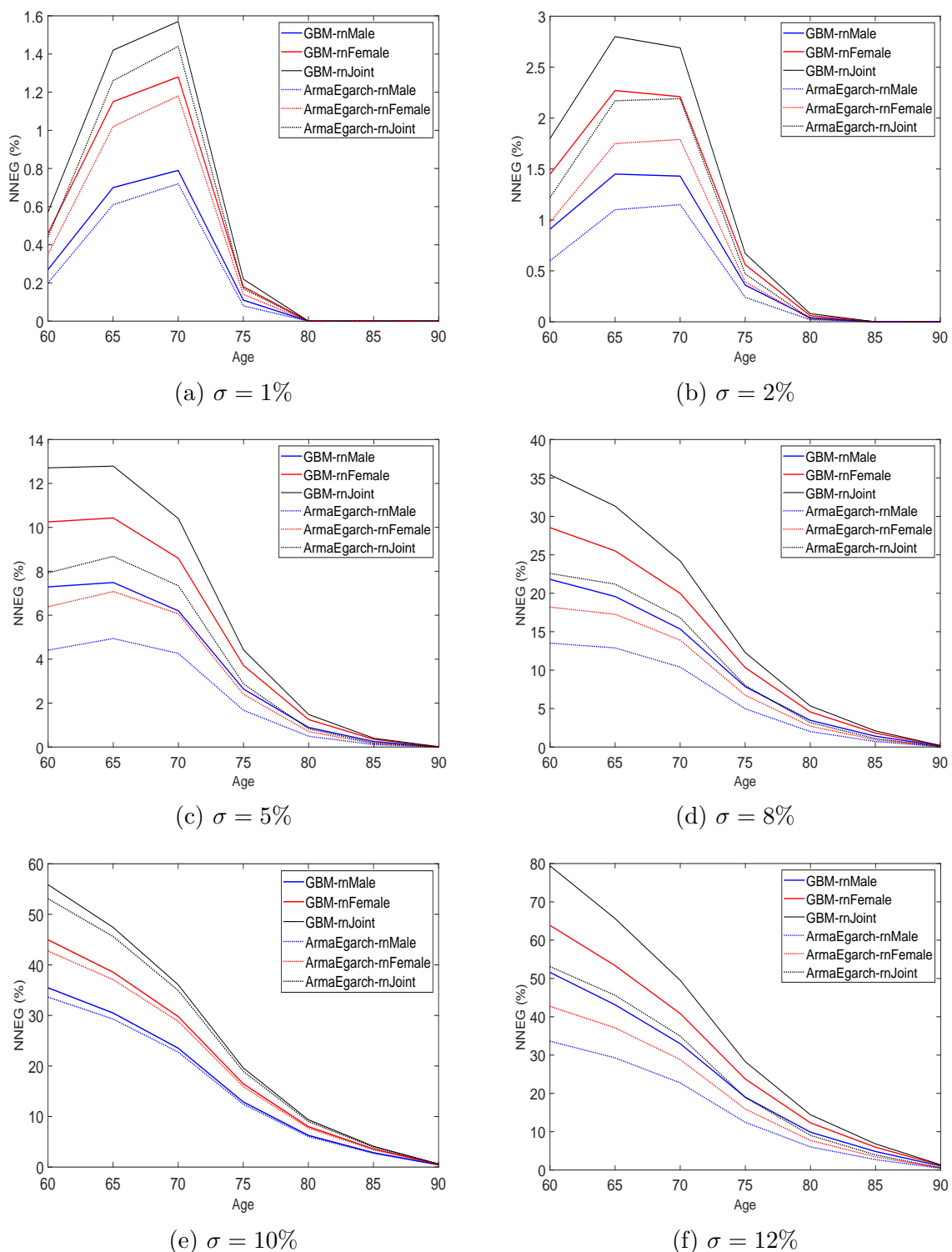


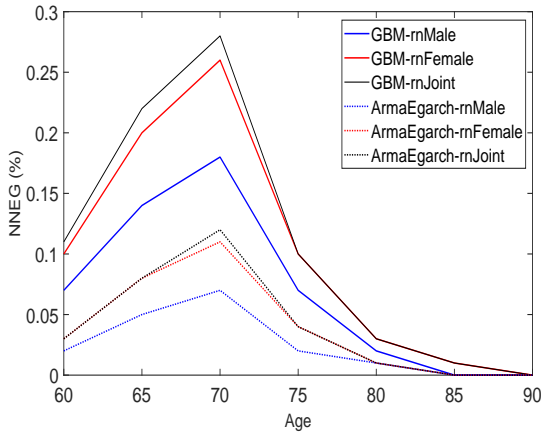
Figure 18: Sensitivity Analysis of NNEG valuation w.r.t. g under baseline loading and $r = 1.75\%$, $R_2 = 5.25\%$ and $g = 1\%$.

It can be noticed that the NNEG values increase with the increase in volatility and

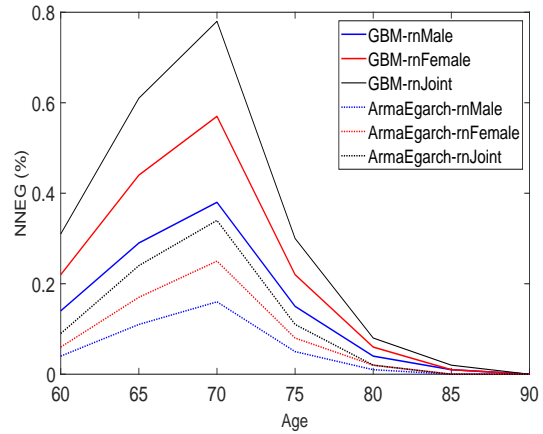
decrease with the decrease in volatility. Doubling the volatility level seems to increase the NNEGs by about 75%. The NNEG values increase almost 7 times fold when switching from $\sigma = 5\%$ to $\sigma = 12\%$.

5.6 Sensitivity to Multiple Decrement Drivers

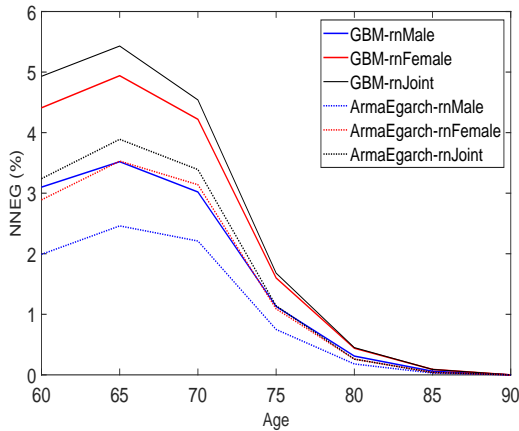
5.6.1 NNEG sensitivities with respect to changes in mortality rates



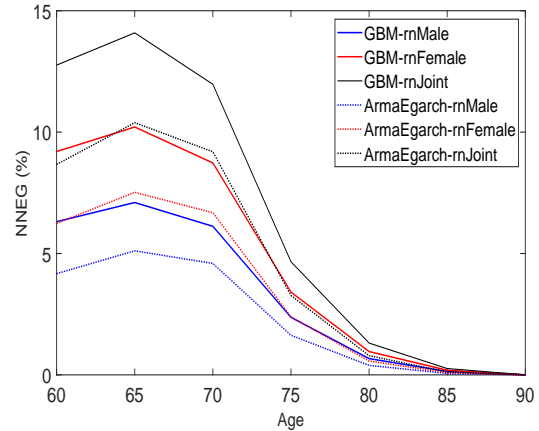
(a) $R_1 = 4.15\%$ and mortality up 20%



(b) $R_1 = 4.15\%$ and mortality down 20%



(c) $R_2 = 5.25\%$ and mortality up 20%



(d) $R_2 = 5.25\%$ and mortality down 20%

Figure 19: Sensitivity Analysis of NNEG valuation w.r.t. changes in mortality, for Flexible LTV baseline loading, $r = 1.75\%$, $\sigma = 3.90\%$.

Here we analyse the effect of ramping up mortality rates by 20% or slowing down mortality rates by 20% for each of the two baseline scenarios. Increasing the mortality rate will bring forward the termination of the loans, which in turn will diminish the NNEG values because the front months values are weighted with larger multiple decrement probabilities. When the mortality rates decrease the NNEG values at the back end are weighted with higher multiple decrement probabilities, and so the total NNEG values will be higher than the baseline scenarios. The impact of 20% increase/decrease of mortality rates on

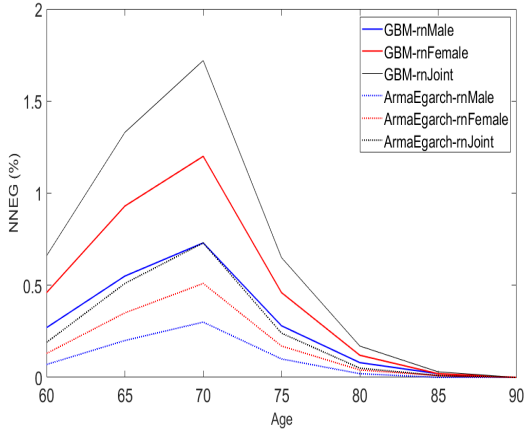
NNEG values is quite small. In order to observe larger changes, very large changes in mortality rates must occur.

5.6.2 NNEG sensitivities due to prepayment rates

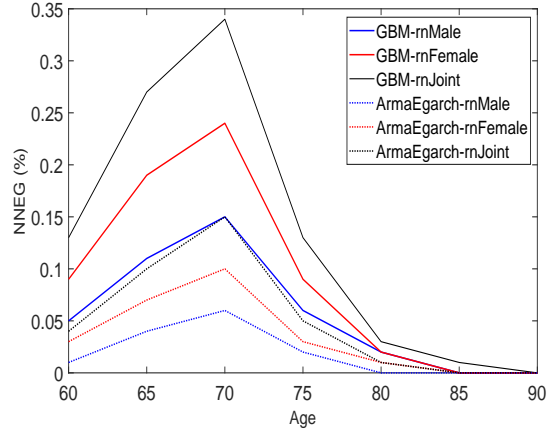
Here we analyse the effect of ramping up prepayments by 20% or slowing down prepayment by 20% for each of the two baseline scenarios. We also included the scenario of no prepayments and the scenario when the prepayments increase by 200%. Overall, prepayment rate changes do not change very much the NNEG profile. Although the NNEG profile does not change much as prepayment rates change, the value does when prepayments change significantly.

From discussion with few insurers, the current evidence on prepayments is rather mixed. For older vintages the prepayment rates are virtually zero. For younger vintages the prepayment rates used for risk-management are higher than those reported previously in Hosty et al. (2008). From discussion with insurers, the prepayment due to refinancing is farther from full capacity. The main driver for prepayment seems to be downsizing when one member of the couple, usually the husband, dies and the surviving borrower decides to move into a smaller property.

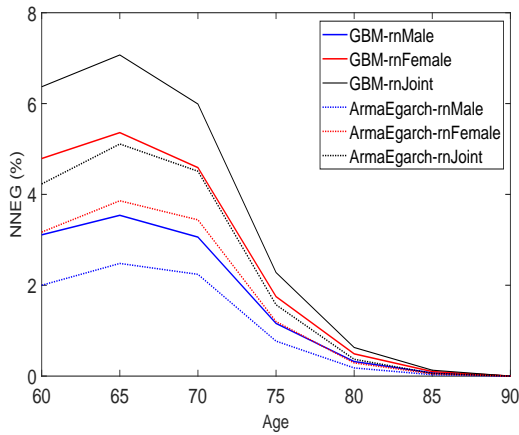
Prepayment rates should increase in economic times characterised by recessions with decreasing interest rate regimes following booming periods with high interest rate regimes. For NNEGs, high prepayments and decreasing roll-up rates are offset by low risk-free rates r . Prepayments should be of concern when risk-free rates are low and roll-up rates are high, that is in the aftermath of a crisis such as the subprime crisis. However, most lenders will try to ramp-up their portfolios in those times and ERM borrowers will not switch shortly after getting their loan due to ERCs and other psychological factors. In the aftermath of a crisis one may expect house prices to be low which will reduce the incentive for refinancing due to the LTV constraint.



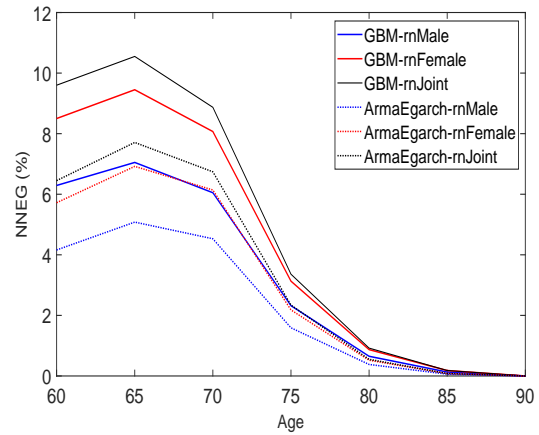
(a) $R_1 = 4.15\%$ and prepayment down to 0%



(b) $R_1 = 4.15\%$ and prepayment up to 200%



(c) $R_2 = 5.25\%$ and prepayment up to 20%



(d) $R_2 = 5.25\%$ and prepayment down to 20%

Figure 20: Sensitivity Analysis of NNEG valuation w.r.t. changes in prepayment, for Flexible LTV baseline loading, $r = 1.75\%$, $\sigma = 3.90\%$.

6 Implied Volatilities

One may consider using the Black-Scholes formula to obtain the implied volatilities from the ARMMA-EGARCH NNEG values.

Here we exemplify how this can be done using our second baseline scenario. There is one put option corresponding to each of the annual maturities. For example, a 60 year old borrower will have 40 maturities to calculate conditionally the NNEG so there are in theory 40 put options. Each option value will be reverse engineered through the Black-Scholes formula to obtain the “equivalent” implied volatility.

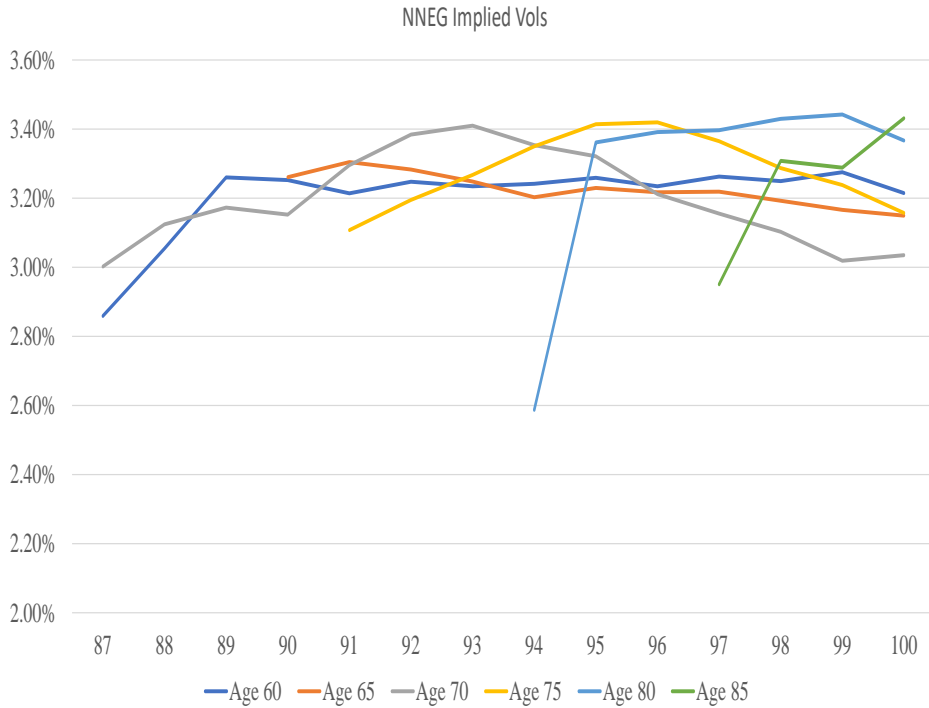


Figure 21: Implied vols term-structures from ARMA(4,3)-EGARCH(1,1) model prices under second baseline scenario and $g = 1\%$.

Figure 21 displays the implied vols term-structures from ARMA(4,3)-EGARCH(1,1) model prices under second baseline scenario. Only calculations for non-zero NNEGs were reported. There is clear variation up and down for the implied volatilities, which is consistent with evolution of volatilities under ARMA-EGARCH. The implied volatilities are annualised and the relatively smaller values are correct since they will drive the growth of the variance of house price returns linearly with time. Thus, in order to arrive at the required level under ARMA-EGARCH model, the GBM volatility should be set-up lower.

Once again, the levels of the implied volatilities seem to suggest that values such as 11% or 13% may be used as stress values and not as current volatility markers.

Recalculating the implied volatilities under the assumption $g = 2\%$ we get the term-structures of implied volatilities in Figure 22. The overall level is not very much different from the one seen in the previous figure, but one can notice a distinct downward sloping pattern.

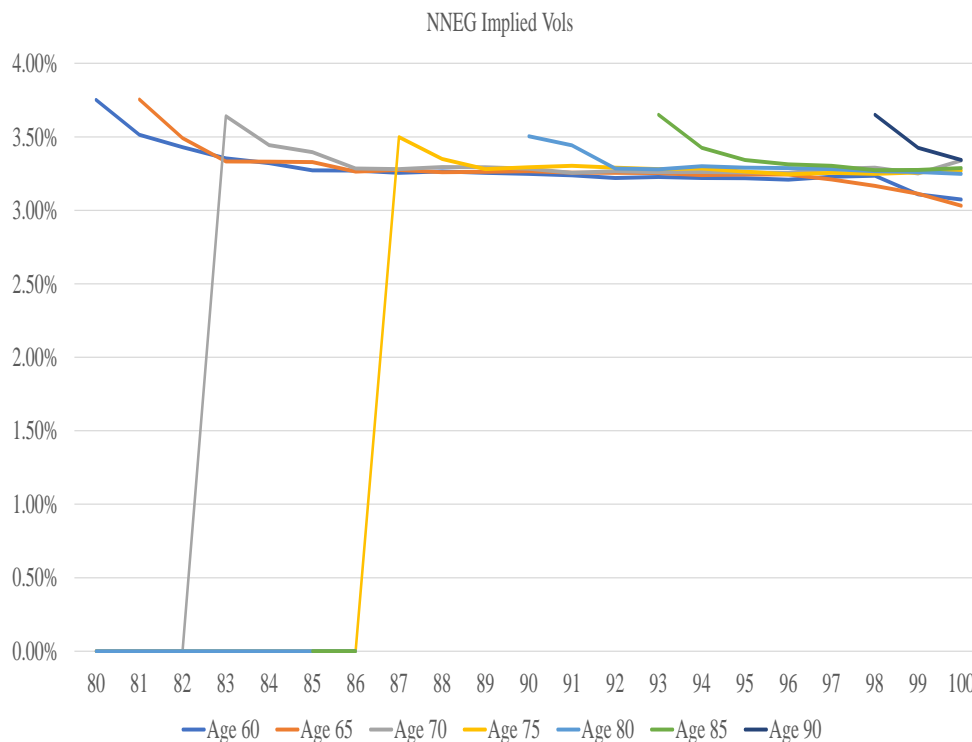


Figure 22: Implied vols term-structures from ARMA(4,3)-EGARCH(1,1) model prices under second baseline scenario with $g = 2\%$.

Some market practitioners believe that for shorter durations, the presence of serial correlation might mean that the BS volatility needed to be higher than historic volatility in order to fit an ARMA result that did pick up that serial correlation. It is not clear to me how increasing volatility for a process that does not have serial correlation, such as the GBM, will induce or recover values as if serial correlation existed. This is an ad-hoc procedure that does not have, to the best of my knowledge, any grounding into statistical modelling.

7 Deferment Rate

7.1 PRA condition

The deferment rate has been introduced adjacently to the rental yield concept with the aim to pinpoint some hard boundary within the NNEG valuation process. Before defining the concept of deferment rate, we should recall the concept of rental yield. The latter has been defined as the ratio of rental income to the price of the property. In Section 4.4 we

described an approximate estimate based on recent rental income data from the Office of National Statistics. Rental yield g plays an important role for fixing the drift under risk-neutral measure for both GBM model and ARMA-EGARCH model.

The deferment rate is key to the fourth principle imposed by the PRA. Their definition is as follows:

By deferment rate, the PRA means a discount rate that applies to the spot price of an asset resulting in the deferment price. The deferment price is the price that would be agreed and settled today to take ownership of the asset at some point in the future; it differs from the forward price of an asset in that the forward price is also agreed today, but is settled in the future.

The deferment rate should not be confused with the rental yield. The former will be denoted henceforth by q , to keep in line with the PRA notation. If H_0 is the house price today the deferment price to get the house at future time T is denoted by $\overleftarrow{F}(T)$ and the deferment rate q is algebraically defined then as

$$\overleftarrow{F}(T) = H_0 e^{-qT} \quad (26)$$

It is evident that $\overleftarrow{F}(T)$ is the prepaid forward price of the house as the underlying asset. Since the forward price is the future value of the prepaid forward price, it follows that

$$F_0(T) = e^{rT} \overleftarrow{F}(T) \quad (27)$$

and combining the two relationships gives the **equivalent** condition

$$F_0(T) = H_0 e^{(r-q)T} \quad (28)$$

One can recognise now the no-arbitrage formula for forward prices on stock index where the dividend yield has been replaced with the “deferment rate” q .

Computationally, if $r - q < 0$ then $\{F_0(T)\}_{T \geq 0}$ decreases with time to maturity so the forward house price curve will be in backwardation. Vice versa, if $r - q > 0$ then $\{F_0(T)\}_{T \geq 0}$ increases with time to maturity so the forward house price curve will be in contango.

The PRA condition via one of the four principles is requiring $\overleftarrow{F}(T) < H_0$ which from (26) is equivalent to ask that $q > 0$. At the same time, the same condition is equivalent to $F_0(T) < H_0 e^{rT}$. In financial economics terms, the condition says that the forward curve on house prices will be bounded by the current house price inflated at the risk-free rate. There are at least two objections one can raise against the approach based on deferment rate. Based on (28), for NNEG put valuations, the guarantee gets in the money most likely when $r - q < 0$. Because of LTV protection, it will less likely be in the money when

$r - q > 0$. For risk-management calculation purposes then, it is very important to have an accurate measurement of q . Lack of data availability and long-term horizon makes this exercise extremely difficult, if not practically impossible. Note that leasehold data refers to give up (hypothetically) the property at time T in the future and the financial economics gains and losses associated with a leasehold are not the same as those for a prepaid forward contract. Prudential Regulation Authority (2018b) is recommending a rate of $q = 1\%$. Interestingly, this is close to the rental yield g we estimated from the rental income data from the Office of National Statistics.

Secondly, the idea that the prepaid forward price should *always* be lower than the current house price can be challenged. This condition will work obviously in normal market conditions and for shorter maturities. However, in the aftermath of financial and economic crises, conditions may be reversed. For example, in the aftermath of the subprime crisis house prices dropped significantly. The usual question of “how much should you pay to get a house in five or ten years time?” should be replaced with the question “what price *can you get* on the market to sign now for possession of a house in five or ten years time?” Even if the house price market was depressed in the aftermath of the subprime crisis, the expectation of the market would naturally be that the market will recover after some time and the forward curve will be in contango. Thus, it is possible that the market will require a prepaid forward that is *higher* than the current house price.

The identity (28) can be rearranged as

$$\frac{F_0(T)}{H_0} e^{-rT} = e^{-qT} \quad (29)$$

so to test whether $q > 0$ we can use data on the left side quantities and see if

$$\frac{F_0(T)}{H_0} e^{-rT} < 1 \quad (30)$$

If on the contrary $\frac{F_0(T)}{H_0} e^{-rT}$ is greater than 1 then this is evidence that $q < 0$. We shall call $\frac{F_0(T)}{H_0} e^{-rT}$ the deferment condition term (DCT). Hence $DCT > 1$ is equivalent to $q < 0$. Remark from 30 that a negative r automatically increases the possibility that $q < 0$, which is consistent with economic periods of deep recession or depression.

In Figure 23, we plot the deferment conditions calculated for the EUREX IPD futures (first five year maturities). The contracts do not correspond to yearly forward contracts, they are futures with annual maturity on a December roll, but they suggest what is plausible to happen in the real estate market. Over the period of this example, in the aftermath of the subprime crisis, although most of the time there is clear evidence that $q > 0$, there is also clear evidence that $q < 0$ for some maturities, and for some maturities quite for a sustained period.

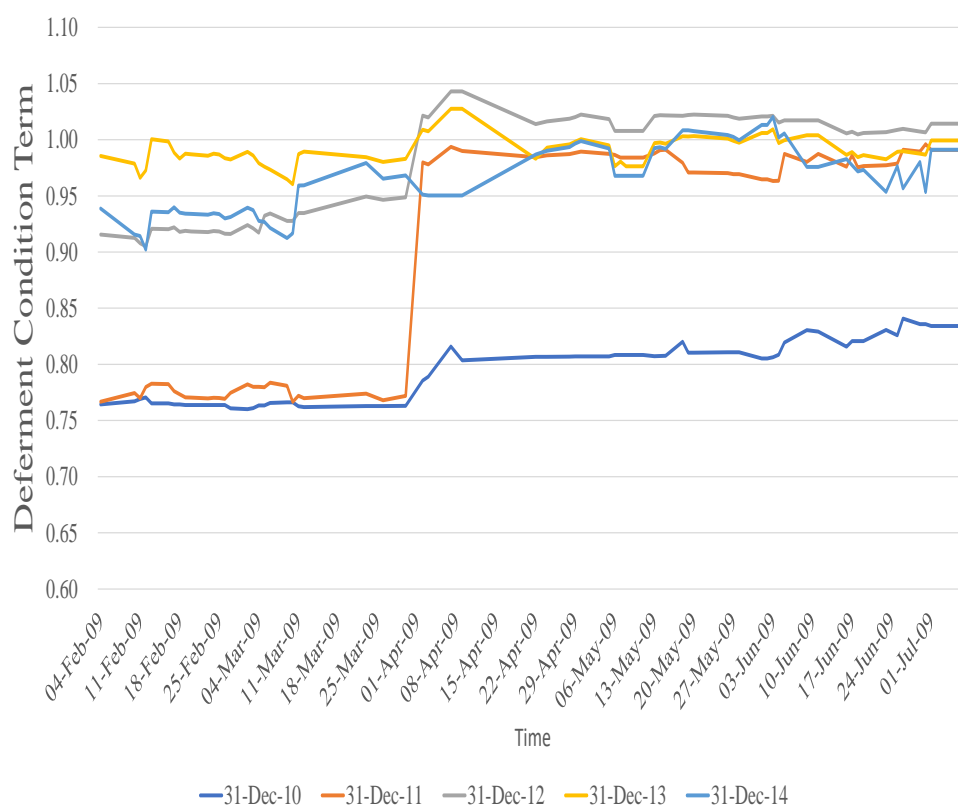


Figure 23: Deferment ratio condition for EUREX futures contracts for the period 4 Feb 2009 to 7 Jul 2009, for all existent five IPD futures contracts.

The futures contracts may be strongly influenced by liquidity, particularly in periods of low market liquidity such as the period illustrated in Figure 23. However, a similar conclusion results from the likewise analysis based on the implied forward prices extracted from the series of total-return-swaps (TRS) traded over the counter on the same underlying IPD index. The data for both analyses has been discussed more amply vis-à-vis potential arbitrage in Stanescu et al. (2014). The graphs in Figure 24 show that during 2009, over a long period of time the DCT for the December 2012 maturity forward was above 1, implying a negative deferment rate.

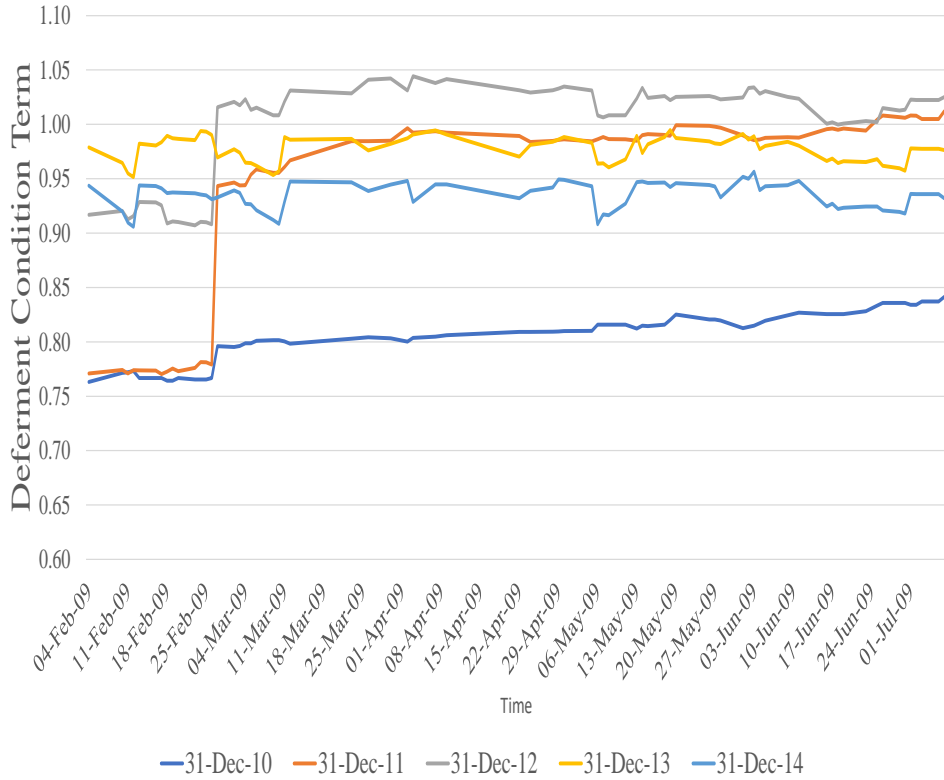


Figure 24: Deferment ratio condition for EUREX futures contracts.

7.2 Arbitrage-Free Bounds *if* Forwards on House Prices Existed

Given the high level of model risk for this asset class it may be useful to have some arbitrage-free bounds that will enforce some market-wide risk-management. The framework described here is inspired by Syz & Vanini (2011). Hence, consider H_t be the value of a property asset or portfolio or index on a total return basis at time t , and consider that there is a forward contract on S_t with maturity T traded with price $F_t(T)$.

We take into consideration r the risk free interest rate, transaction costs k_b or k_s for buying or selling a property respectively, defined for a one way transaction in percentage terms.

For a given investment horizon $T - t = \tau$, an investor buys a property portfolio H_t by borrowing the proceeds, including associated friction costs, $H_t(1 + k_b)$. Simultaneously, the investor enters a short forward position in the amount of $F_t(T)(1 - k_s)$.

At the end of the investment horizon, the investor sells the property portfolio for $H_T(1 - k_s)$ and also settles the short forward contract $F_t(T)(1 - k_s) - F_T(T)(1 - k_s)$ and the loan

with interest, $H_t(1 + k_b)e^{r\tau}$. To avoid arbitrage

$$F_t(T)(1 - k_s) - H_t(1 + k_b)e^{r\tau} \leq 0. \quad (31)$$

or equivalently

$$\boxed{F_t(T) \leq \frac{(1 + k_b)}{(1 - k_s)} H_t e^{r\tau}.} \quad (32)$$

On the other hand, *if* an arbitrageur could sell a property portfolio short¹⁹ in the amount $H_t(1 - k_s - k_3)$, where k_3 represents the value of the possibility to perform a short sale, that is $k_3 = 1 - k_s$ if short-selling is not allowed and $k_3 = 0$ if full short-selling is possible, the investor then could enter a long forward position in the amount of $F_t(T)(1 + k_b)$. The combined positions at maturity T gives

$$F_T(T)(1 + k_b) - F_t(T)(1 + k_b)$$

from the long forward contract and

$$H_t(1 - k_s - k_3)e^{r\tau}$$

from the short-sale position. The joint pay-off will allow the investor to buy property in the amount of $H_T(1 + k_b)$ and to close the short position. To avoid arbitrage, since $F_T(T) = H_T$, we get

$$H_t(1 - k_s - k_3)e^{r\tau} - F_t(T)(1 + k_b) \leq 0. \quad (33)$$

or equivalently

$$\boxed{H_t e^{r\tau} \frac{(1 - k_s - k_3)}{(1 + k_b)} \leq F_t(T)} \quad (34)$$

Combining (32) and (34) gives

$$H_t \frac{(1 - k_s - k_3)}{(1 + k_b)} e^{r\tau} \leq F_t(T) \leq H_t \frac{(1 + k_b)}{(1 - k_s)} e^{r\tau} \quad (35)$$

Considering ρ_τ as the convenience yield closing the equation

$$F_t(T) = H_t e^{(r + \rho_\tau)\tau} \quad (36)$$

we find by reverse engineering the lower and upper arbitrage-free bounds of the conve-

¹⁹This may be possible fractionally using a cross-hedging asset, perhaps?!

nience yield

$$\rho_\tau^U = \frac{1}{\tau} \ln \frac{(1 + k_b)}{(1 - k_s)} \quad (37)$$

$$\rho_\tau^L = \frac{1}{\tau} \ln \frac{(1 - k_s - k_3)}{(1 + k_b)} \quad (38)$$

Syz & Vanini (2011) showed how one can calibrate k_b, k_s, k_3 to over-the-counter house price derivatives on the Halifax index, endogenously. The values of these parameters can be also taken exogenously from market information.

From (32) we can see that an upper bound for the prepaid forward price is

$$\overleftarrow{F}(T) \leq H_0 \frac{1 + k_b}{1 - k_s} \quad (39)$$

Therefore, if k_b is the transaction cost for buying the property (stamp duty) and k_s is the cost for selling the property the bound is given by $\frac{1+k_b}{1-k_s}$ which is always greater than 1. This allows for both economic scenarios $\overleftarrow{F}(T) < H_0$ and $\overleftarrow{F}(T) \geq H_0$ to coexist.

As of 2018, the stamp duty rate in the UK is 2% for properties (or portions) between £125,000-250,000, 5% for properties (or portion) between £250,001-925,000, 10% for properties (or portion) between £925,001-1.5mil. and 12% above that. For exemplification we shall consider a property worth £275,000 so then the stamp duty is calculated as follows: 0% on the first £125,000, 2% on the next £125,000 = £2,500, and 5% on the final £25,000 = £1,250. Hence the stamp duty is equal to £3,750 that gives $k_b = 1.3636\%$. In 2018, the average real estate agents' commission was between 1% and 3% (including VAT). Here we take $k_s = 2\%$. That would give a boundary constant equal to 103.4323% . Hence the prepaid forward price should be smaller than $1.0343 \times H_0$. Transforming this boundary constant into a deferment-type rate (for one year ahead)

$$\widetilde{q}^* = -\ln\left(\frac{1 + k_b}{1 - k_s}\right)$$

gives $\widetilde{q}^* \approx -3.3747\%$. The above example should be interpreted in an approximate way since legal costs, moving house costs, and other hidden costs are not represented.

8 Discussion of Results

The current results point out to some early important points.

- In the absence of market prices or recognised benchmark prices, it is difficult to identify the best model (method) with reference to ERM prices. The best that can be done under current circumstances is to (a) look for a model that has good

forecasting power of house prices; and (b) compare various models across a large set of scenarios, from standard baseline to stressed scenarios.

- ARMA-EGARCH family of models outperforms the GBM model under real-world measure in terms of forecasting short- and medium-term house prices. This is not surprising since the theoretical properties of the GBM model are in contradiction with the empirical stylised features of house price time series.
- The GBM (Black-Scholes) model is simple to implement but it lacks theoretical support for this asset class. It may inflate in relative terms the NNEG for the young age borrowers due to high variance of house prices at long maturities.
- The method of parameter estimation may give different results. For GBM the GMM parameter estimation method may produce superior forecasting results versus MLE and MM. At the same time, the estimates under GMM can be substantially different from the parameter values estimated by MLE or MM.
- Black 76 model theoretical formula for pricing European options is based on the futures of the underlying asset- house prices in this case. In the absence of a residential house futures contract, one cannot switch modelling from spot house prices onto futures.
- Moreover, one common mistake in some papers covering NNEG valuation is to refer to the forward house prices in relation to the Black 76 model. In addition, if the forward house prices are produced using a formula that has not been proved from the first principles, the risk management for ERMs may become unreliable.
- Using multiple decrements will always deflate NNEG values due to earlier termination. In this study, we reported only multiple decrement results, but single decrement results are also available from the authors upon request.
- It is possible to get similar NNEG vectors under very different models. Adjusting volatilities (one value for each maturity) under GBM-rn may lead to a matching of NNEG values produced by an ARMA-EGARCH model.

Our analysis shows that NNEG values produced by the GBM cannot be considered lower bounds for NNEGs calculated from more appropriate models, nor upper bounds. The only exception is the regions of high NNEG risk when the two classes of models (GBM on one part and the ARMA-EGARCH on the other) come out almost the same.

The valuations under ARMA-EGARCH models may look more complicated computationally but theoretically they are more robust. There is still a question about what is the best way to risk-neutralise the results. The current method of risk-neutralisation for

the ARMA-EGARCH using the conditional Esscher transform still makes use of the $r - g$ in the martingale calibration and this is slightly concerning.

Further research beyond the timeline of the current study may consider a two-factor mean-reverting diffusion model, with one factor modelling house prices and a second factor capturing rental yields.

In the next stage of the research, the following other issues may be considered:

- enlarge the set of models by adding discount curves resulting from stochastic models
- consider how to deal with idiosyncratic house price risk
- include mean-reverting models and jump-diffusion models for house prices
- consider extreme economic scenarios similar to the ones used in rating agencies methodologies for ERM securitization
- consider the portfolio valuation and the associated cash-flows including various costs of funds

These further research ideas will be explored in the near future depending on funding becoming available.

Notations and Glossary of Terms

The following list of notations are used in this review:

g continuously compounded rental yield

H_t is the house price index at time t

K_t is the accumulated loan balance, usually equal to $K_t = L_0 e^{Rt}$

L_0 is the initial loan value

μ is the expected growth rate for house price returns under GBM model

η is the limit total number of months to be considered

ν represents the percentage giving the LTV ratio

$q(t)$ denotes the ERM loan survival probability at time t

R is the roll-up rate charged on the loan; this is the rate at which the loan balance grows

r is the risk free discount rate

σ is the volatility parameter for the house price series

Glossary OF Terms

ARMA-EGARCH: autoregressive moving average exponential autoregressive conditional heteroskedasticity model

ARMA-EGARCH-rn: refers to the ARMA-EGARCH model under the risk-neutral measure.

ARMA-EGARCH-rw: refers to the ARMA-EGARCH model under the real-world measure.

Black76: the Black-Scholes variant referring to pricing European options on futures

conditional Esscher martingale measure: it is a risk-neutral measure constructed in a specific way when we are in incomplete markets

ERC: early repayment charges

GBM: geometric Brownian motion

GBM-rn: geometric Brownian motion under risk neutral measure

GMM: generalized method of moments

Incomplete market: it is a market where a derivative product cannot be replicated from portfolios of primary traded assets defining the market

LTV: loan to collateral house value ratio

LTC: long term care risk

MLE: maximum likelihood estimation

MM: method of moments

Morbidity rate: it refers to the rate of borrowers moving into long-term care

Multiple decrements probability: the probability of termination of contract due to either mortality, long-term care or prepayment

NNEG: non-negative-equity guarantee

Roll-up rate: the roll-up rate charged on the loan; this is the rate at which the loan balance grows.

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9 Appendix A

9.1 Mortality Table Office for National Statistics

Table 11: Mortality Table Office for National Statistics 2015-2017

Age	Male	Female
x	qx	qx
55	0.56%	0.4%
56	0.61%	0.4%
57	0.61%	0.4%
58	0.68%	0.5%
59	0.75%	0.6%
60	0.85%	0.6%
61	0.96%	0.6%
62	1.02%	0.7%
63	1.12%	0.8%
64	1.22%	0.8%
65	1.26%	0.9%
66	1.47%	1.0%
67	1.64%	1.1%
68	1.58%	1.1%
69	1.89%	1.3%
70	2.09%	1.4%
71	2.28%	1.5%
72	2.57%	1.7%
73	2.72%	2.0%
74	3.18%	2.1%
75	3.53%	2.2%
76	3.82%	2.6%
77	4.27%	2.9%
78	4.60%	3.3%
79	5.42%	3.6%
80	5.72%	4.2%
81	6.38%	4.5%
82	7.21%	5.3%
83	7.89%	6.0%
84	9.07%	6.8%
85	10.42%	7.8%
86	11.49%	8.9%
87	12.06%	9.6%
88	14.22%	11.2%
89	15.38%	12.6%
90	17.01%	14.2%
91	20.26%	16.3%
92	20.79%	17.5%
93	22.32%	19.9%
94	25.25%	22.1%
95	27.05%	23.9%
96	28.71%	25.4%
97	30.49%	27.5%
98	35.92%	28.3%
99	38.74%	31.4%
100	34.60%	32.8%

10 Appendix B

Here are some assumptions previously used in the literature.

10.1 General characteristics

The minimum age for ERMs is 55/60/65. In Italy, the minimum age by law is 65. Dowd (2018) takes 70 as the base case scenario.

Regarding costs, Hosty et al. (2008) reported the following policy expenses: administration (initial GBP500; in force GBP60; termination GBP350), distribution and sales 2.5% of the customer advance, marketing 1% of customer advance.

Ji (2011) used for the UK the average time delay for house price sale $\delta = 0.5$ year, cost for selling the property $c = 2.5\%$ of the value of property.

10.2 Rental yield

Ji (2011) used for UK the rental yield $g = 2\%$ while Dowd (2018) mentions $g = 2\%$ and $g = 3\%$ as a base case rate, increasing to $g = 4\%$ as a stress test, and varying between 1% , 0% and -2.75% as well. Hosty et al. (2008) used 3.3% .

10.3 LTV

Table 12: Loan to values (LTVs) for various equity release mortgages issued 29/11/2018. Source: Legal & General

Age	Flexible	Flexible Plus	Flexible Max	Flexible Max Plus
55	11.50%	16.00%	21.20%	24.00%
56	12.50%	17.00%	22.40%	25.00%
57	13.50%	18.00%	23.60%	26.00%
58	14.50%	19.00%	24.80%	27.00%
59	15.50%	20.00%	26.00%	28.00%
60	17.00%	21.00%	27.50%	29.50%
61	18.00%	22.00%	28.50%	31.00%
62	19.00%	23.00%	29.50%	32.00%
63	20.00%	24.00%	30.50%	33.00%
64	21.00%	25.00%	31.40%	34.00%
65	22.50%	26.50%	32.20%	35.10%
66	24.00%	28.00%	32.50%	36.20%
67	24.80%	29.00%	33.50%	37.30%
68	25.60%	30.00%	34.50%	38.40%
69	27.00%	31.50%	35.50%	39.50%
70	28.50%	33.00%	36.60%	41.10%
71	29.70%	34.00%	37.70%	42.20%
72	30.50%	35.00%	39.00%	43.40%
73	31.20%	35.50%	40.00%	44.60%
74	31.70%	36.00%	41.00%	45.80%
75	32.40%	37.00%	42.00%	47.00%
76	33.20%	38.00%	43.00%	48.00%
77	34.00%	39.00%	44.00%	49.00%
78	35.00%	40.00%	45.50%	50.00%
79	35.50%	41.00%	46.50%	50.50%
80	36.50%	42.00%	48.00%	51.50%
81	37.50%	43.00%	49.00%	52.50%
82	38.50%	44.00%	49.40%	53.00%
83	39.50%	45.00%	49.80%	53.00%
84	40.50%	46.00%	50.20%	53.00%
85 and over	41.50%	47.00%	50.50%	53.00%

For the LTV we also point to 40% (Dowd 2018), to 27% for new drawdowns and 32% for new lump sum plans as reported in the Equity Release Council (ERC) 2017 Report.

Hosty et al. (2008) is using an initial loan advanced as £20,000 while LTV starts from 15% at age 55 and increases by 1% each year up to 50% at age 90. The minimum house price is £70,000. This procedure sets the maximum house price²⁰ to 133,333 at age 55. Similarly, Li et al. (2010) has a minimum house price of £60,000, starts from 17% at age 60 and increases by 1% each year up to 50% at age 90 and an initial loan at £30,000. An overall average value of 20% seems to be representative, but a more refined table taken into account age is also useful.

10.4 Interest Rate Risk

The most evident risk affecting ERMs is interest rate risk. Given the long and uncertain maturity of these loans, one needs to rely on models to simulate future paths for interest rates. Lenders of ERMs use, in general, two types of rate. The rate R is the rate charged on the loan. This is the rate at which the loan balance grows. Secondly, there is the discount rate $\{r_t : t \geq 0\}$ which is the discount rate used to calculate the present value of the mortgage loan. Very often in the literature $r_t \equiv r$, which is a constant risk-free rate used for discounting purposes. There is evidence that, where choice is available, borrowers will prefer adjustable rates to fixed rates.

From a lender perspective, Cho et al. (2013) advocated using a multi-period cash-flow model incorporating house price risk, interest rate risk and termination delay. They argue that the lump sum mortgages are more profitable and less risky than the tenure ERMs. One possible explanation is that the analytical valuation of an ERM with tenure payments is far more complex²¹ than that for a lump sum mortgage.

A very interesting observation (Pfau 2016) linked to interest rates is how the line of credit of an ERM grows. The loan balance typically grows at a rate given by the reference interest rate, say one-month LIBOR, a fixed spread reflecting the lender's profit margin and plus a fixed mortgage insurance premium. The sum rate is called the effective rate and is applied to project the growth of the loan balance. The same rate is also applied to increase the overall principal limit, which for line-of-credit ERM contracts is equal to the balance of the line-of-credit plus the loan balance and plus set-asides. The design arbitrage is that interest and insurance premiums are charged only to the loan balance. The line-of-credit and set-aside accrue under the effective rate *as if* these rates are also charged to these ledgers.

Li et al. (2010) assumed a risk-free continuously compounded rate $r = 4.56\%$ that is the average yield from the 20-year nominal zero-coupon British government securities in the year 2007; and a roll-up rate $R = 6.39\%$ continuously compounded obtained as the average roll-up rate for the top 10 UK equity release providers in May 2007. Ji (2011)

²⁰There seems to be a typo in Hosty et al. (2008) who give 233,333.

²¹A valuation framework that takes into consideration the mortality risk, interest rate risk and housing price risk is detailed in Lee et al. (2012).

used for the UK the following parameters for NNEG valuation: $r = 4.75\%$; rate $R = 7.5\%$ while Dowd (2018) takes as the base case $r = 1.5\%$ (and decreasing to 0.5% for a stress scenario).

For the Korean market, Lew & Ma (2012) reported that the average value of the 10-year government bond rates was 5.12% between 2002 and 2007 so the expected interest rate was calculated as 7.12% after adding 2% lender's margin. Those values were adjusted in Feb 2012 to be 3.3% for house prices and 6.33% for the expected interest rate.

Some articles assume independent evolution between house prices and interest rates, Chinloy & Megbolugbe (1994), Wang et al. (2008). Others assume a two-factor model correlating house prices and interest rate dynamics (Huang et al. 2011), or a multidimensional regression model as in Chang et al. (2012) and a VAR approach as in Alai et al. (2014).

The interest rates can be fixed, and many borrowers seem to prefer this route, but it can be very steep, in some cases the rates being in double digit figures such as 12% or 15% . Annually-adjustable rates can be used to link the payments on the ERMs to a reference interest rate. The reference rates that have been used on the market are the 1-year constant maturity treasury, the 1-month and 1-year LIBOR, the 10-year Treasury rate in the USA, and the certificate of deposit (CD) rate in Korea. In order to avoid liquidity pressures, this rate is usually not allowed to vary by more than few percentage points within a year.

10.5 Longevity or Mortality Risk

The sellers of ERMs have considered for a long time that longevity risk is diversifiable. Hence, by pooling a large numbers of loans we could use mortality tables to determine the terminations of loans. The same idea applies to long-term care risk and prepayment risk.

The mortality data used in Hosty et al. (2008) is derived from the Continuous Mortality Investigation Research (CMI "00") mortality tables. The tables are referred to as Immediate Annuities Male Lives (IML "00") and the Immediate Annuities Female Lives (IFL "00"), adjusted for cohort effects (i.e. where rates of improvement in mortality have been different for people born in different periods historically). The tables show the probability of death during any year for an individual of a particular age who is alive at the start of that year.

Hosty et al. (2008) discussed how to adjust mortality rates for different socio-economic classes and by property value. Table 14 shows the adjustment factors that occur due to different socio-economic conditions while Table 15 indicates the adjustment factor by the type of property.

Table 13: Longevity expectations based on Immediate Annuities Male and Female Lives.

Year	Expectation of life at birth		Expectation of life at age 65	
	Male	Female	Male	Female
1841	40	42	11	12
1900	49	52	11	12
2000	76	80	16	19
2020	79	83	18	21

Notes: Derived from Continuous Mortality Investigation Research 00 tables.

Table 14: Mortality of different socio-economic classes as a percentage of population mortality: Source Hosty et al. (2008).

Class	Ages 50-64
I	72%
II	77%
IIIN	104%
IIIM	130%
IV	120%
V	180%

Table 15: Mortality assumptions by property value

Property Value	Mortality Assumption
up to GBP130k	120% base
GBP130k -GBP250 k	100% base table
GBP250 k-GBP750 k	85% base table
GBP750k +	55% base table

As an additional stress scenario, Dowd (2018) considers the expected (mean) longevity increased by two years.

One may also use the T08 series of term mortality tables, based on 2007-2010 data collected by the CMI. We use the Office for National Statistics mortality tables (ONS tables for 2015-2017).

10.6 Joint Mortality Modelling

In many instances the loan is given to a living couple. The loan will survive as long as one of the couple survives. One common assumption is to use for a borrowing couple a 95% adjustment factor of the base mortality table for the male and female.

Knapcsek & Vaschetti (2007) calculate the joint *cumulative* probability of death after j years for a couple $(1, 2)$ with the formula

$$P_{12}(j) = P_1(j) \times P_2(j) \tag{40}$$

where $P_i(j)$ is the cumulative probability of death by year j for the partner i , with $i = 1, 2$. There is also a possible correlation built-in as couples can take care of each other and survive longer.

10.7 Long Term Care Risk

When premiums were originally set for the HECM²² loans, there was no actual exit data so the assumption made was that loan exits would occur at 1.3 times the rate of mortality, see Rodda et al. (2004). The actuarial market practice in the UK calculates morbidity as a factor of the mortality rate.

Table 16: Percentage loading to base mortality due to long term care entry: Source Hosty et al. (2008).

Age	Male(%)	Female(%)
≤ 70	2	3
(70, 80]	4	12
(80, 90]	5	13
(90, 100]	4	8

For multi-state modelling considering the interaction between long-term care entry and mortality is paramount because there is significantly higher mortality experienced by long-term care residents compared to “at home” mortality means that to maintain the same aggregate assumption for mortality by age lighter than average mortality should be assumed for “at home” lives. Table 16, from Hosty et al. (2008), shows the long-term care net impact of additional decrements, offset by reductions in at-home mortality, taken to be the uplifts to base mortality, with intermediate values by linear interpolation.

10.8 House Price Risk

Although CPI and GDP are perceived as major risk factors for the house price inflation (HPI). However the CPI deflated house price growth and the GDP deflated house price growth vary significantly from country to country.

Ji (2011) employs the following assumption on initial house values, as detailed in Table 17.

Table 17: Minimum initial house values: Source Institute of Actuaries (2005)

Age of the younger spouse at inception	60	70	80	90
Initial house value	176,500	111,000	81,000	60,000

²²The only reverse mortgage insured by the US Federal Government is called a Home Equity Conversion Mortgage (HECM), and is only available through an Federal Housing Administration (FHA) approved lender. If you are a homeowner age 62 or older and have paid off your mortgage or paid down a considerable amount, and are currently living in the home, you may participate in FHA’s HECM program. The HECM is FHA’s reverse mortgage program that enables you to withdraw a portion of your home’s equity.

Lew & Ma (2012) used a housing price growth rate of 3.5% per annum, reflecting the average house price growth rate in Korea between 1986 and 2006. Moreover, Hosty et al. (2008) argues that the house price inflation growth rate should be between RPI and the economic growth plus a “bit”, and he gives 2.5% to 5.5% as a confidence interval for the house price inflation, “with either extreme difficult to justify”.

10.9 Prepayment Risk

Not very much is known about the values of the prepayment rate for ERMs. In the US in the early days of the HECM programme, a flat prepayment rate of 0.3 times the mortality rate of the youngest borrower in the family was used. In Korea, a prepayment rate of 0.2 times the 2010 mortality rate for females was chosen based on Korean demographic data. Prepayment risk is usually managed with early redemption charges (ERC). Hosty et al. (2008) describe this feature that varies by different providers. The ERC can be fixed rate charge or marked to market. In August 2007, the fixed charge scales ranged from 3% flat for the first 5 years and nil thereafter, to 7% initially stepping down to nil after 10 years and some providers applied charges for the first 20 years. Many large providers were charging mark to market penalties with the ERC applied depending on interest rate movements between inception and repayment. The ERCs were capped (currently at either 20% or 25%).

Hosty et al. (2008) considered the following prepayment rates. The first set was taken from the Norwich Union prospectus for Equity Release Funding (no.5) plc, August 2005, as follows: ERF1, 4.4% p.a.; ERF2, 3.7%; ERF3, 2.5%; ERF4, 1.4% (prepayment rates given by number of loans). In addition, the prepayment rates in Table 18 were noted from Bell & Bain Ltd, Glasgow.

Table 18: Prepayment rates reported in Hosty et al. (2008)

Year	Prepayment rate (%)
1-2	1.00
3	2.0
4-5	2.5
6-8	2.0
9-10	1.0
11-20	0.5
21+	0.25

Ji (2011) separated prepayment rates into two sources, Table 18 for remortgaging of ERMs and Table 19 for prepayment arising from changes in personal circumstances.

10.10 Discount Factors

One issue that is often neglected in NNEG valuation is the choice of discount factors. Quite often the discount factors are derived from a unique constant risk-free rate, accepted

Table 19: Prepayment rates reported in Ji (2011): Source Institute of Actuaries (2005)

Year	Prepayment rate (%)
1	0.0
2	0.0
3	0.15
4	0.3
5	0.3
6+	0.75

in the framework described by Knapcsek & Vaschetti (2007), see also Dowd (2018). Kogure et al. (2014) used $df(t) = (1 + r)^{-t}$ with $r = 0.5\%$ for the Japanese market. Li et al. (2010) use returns from Treasury-bills as a proxy for short-term interest rate and Kim & Li (2017) employed the 91-day certificate of deposit as a proxy for the same risk-free rate. Hosty et al. (2008) used a constant risk-free rate equal to 4.5% but considers the discount rate as 4.75%, effectively extracting the NNEG risk premium by applying the same time invariant risk premium of 0.25%.

Wang et al. (2014) and Lee et al. (2012) employ a CIR short-rate model for discount curves, which will also fit in the framework described in Knapcsek & Vaschetti (2007).

We used in our calculations the risk-free curve on 26 December 2018, downloaded from Bloomberg. This is described in Table 20.

Table 20: GBP Risk-free term structure of interest rates on 26 December 2018.

Maturity	26/12/2018
1M	0.67%
3M	0.73%
6M	0.77%
1Y	0.74%
2Y	0.72%
3Y	0.72%
4Y	0.78%
5Y	0.88%
6Y	0.92%
7Y	0.98%
8Y	1.05%
9Y	1.16%
10Y	1.26%
12Y	1.37%
15Y	1.47%
20Y	1.71%
25Y	1.76%
30Y	1.78%
40Y	1.69%
50Y	1.68%

10.11 Costs of Funds

Hosty et al. (2008) stated the following annualised funding costs based on the information from the wholesale banking markets at the time of their research: average swap rate 5.10%; funder's margin over LIBOR 0.40%, redemption profile insurance and risk

premium 0.25%, cost of solvency capital 0.07%.

The cost of redemption profile insurance is discussed in more detail in Hosty et al. (2008). The idea is that funders of ERMs will buy insurance to rematch earning LIBOR on the full outstanding balance of the portfolio, irrespective if the portfolio level falls below best estimate such as if multiple decrements are faster than predicted, in which case there is a cost of breaking swaps, or if the portfolio level is higher than the best estimate such as in the case of delayed multiple decrements due to cohort behaviour, innovation in medicine etc. in which case more swaps must be added. The financial instrument that helps managing this risk is the balance guaranteed swap (BGS). Hosty et al. (2008) mentions that before 2007 a full cover BGS had a cost of 70 bps p.a. and buyers of BGS usually reduced this hedging costs by using a narrower confidence interval around expected redemption portfolio profile.

The pricing and management of the BGS has been discussed in Fabozzi et al. (2009), Fabozzi et al. (2010) and, more recently, in more detail in Tunaru (2017). The standard pricing is done based on a portfolio of swaptions or amortising swaptions. One problem frequently ignored by BGS market makers is that when a swaption is exercised, the inherited swap, although contributing positively towards hedging the desired risk short term, it may change later on into a liability. Pricing can be done also with a portfolio of caps or with a portfolio of floors, that are more expensive than swaptions but they do not carry any downside.

Another problem here is that the notional is not always amortising or accreting (negative amortisation). The outstanding balance depends on remaining loans, individual loan balance growth, house prices and age of borrowers. Therefore, the BGS price will be more difficult to calculate than the usual BGS price related to forward mortgages.

11 Appendix C

The GBM dynamics is specified under the real-world measure using the equation

$$dH_t = \mu H_t dt + \sigma H_t dW_t \quad (41)$$

For simplicity, we denote by $K = L_0 e^{RT}$ the exercise price of our NNEG put option at maturity T .

11.1 Risk-neutral world GBM pricing

Under risk-neutral world the dynamics changes only in the drift to

$$dH_t = (r - g) H_t dt + \sigma H_t dW_t \quad (42)$$

where g is the rental yield.²³

The Black-Scholes formula behind the NNEG put option is

$$Put(H_0, K, T) = e^{-rT} E^Q (\max[K - H_T, 0]) \quad (43)$$

where Q is the risk-neutral measure implied by the Black-Scholes model. Then

$$\boxed{Put(H_0, K, T) = Ke^{-rT}\Phi(-d_2) - H_0e^{-gt}\Phi(-d_1)} \quad (44)$$

where $d_1 = \frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (r - g + 0.5\sigma^2)T]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

11.2 Real-world GBM pricing

Under this method securities are priced using real-world probabilities derived from the historical information and a risk-neutral (funding rate) discount rate.

This would be valued under real-world measure as

$$Put(H_0, K, T) = e^{-r^*T} E^P (\max[K - H_T, 0]) \quad (45)$$

where r^* should be the risk-adjusted interest rate reflecting the premium charged for investing in this market.

Using the usual trick that

$$\begin{aligned} E^P (\max[K - H_T, 0]) &= E^P \left((K - H_T) 1_{\{H_T < K\}} \right) \\ &= E^P \left(K 1_{\{H_T < K\}} \right) - E^P \left(H_T 1_{\{H_T < K\}} \right) \\ &= KP(H_T < K) - E^P \left(H_T 1_{\{H_T < K\}} \right) \end{aligned} \quad (46)$$

One can show with standard calculations that

$$P(H_T < K) = \Phi \left(-\frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (\mu - 0.5\sigma^2)T] \right)$$

and

$$E^P \left(H_T 1_{\{H_T < K\}} \right) = H_0 e^{\mu T} \Phi \left(-\frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (\mu + 0.5\sigma^2)T] \right)$$

Thus

$$\boxed{Put(H_0, K, T) = e^{-r^*T} \left[K\Phi(-d_2) - H_0e^{\mu T}\Phi(-d_1) \right]} \quad (47)$$

where $d_1 = \frac{1}{\sigma\sqrt{T}} [\ln(H_0/K) + (\mu + 0.5\sigma^2)T]$ and $d_2 = d_1 - \sigma\sqrt{T}$.

²³We consider rental yield here in order to be able to compare GBM-rn as used by some insurers with other approaches. We do not necessarily agree that $g \neq 0$

11.3 Black 76 Model

Some argued that the “correct” approach is to use the Black (1976) formula for pricing the NNEG. Under this model pricing the NNEG would be done with the formula

$$Put = e^{-rT} [K_T N(-d_2) - F(T) N(-d_1)] \quad (48)$$

with

$$d_1 = \frac{\ln(F(T)/K_T) + 0.5\sigma^2\tau}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

where r is the risk-free rate of interest, K_T is the strike price for period T calculated as $K_T = L_0 e^{R \times T}$ (here L_0 is the initial loan value) and $F(T)$ is the *forward* house price for year T , which *also* has the formula

$$F(T) = H_0 e^{(r-g)T} \quad (49)$$

where g is the house rental rate and H_0 is the current house price.

11.4 De-smoothing approach

One approach to deal with serial-correlation in house prices that is apparently being used by life actuaries working on annuities is to use a desmoothing procedure and get the modelling that way. While we do not fully agree with the standard desmoothing procedure that is normally applied to commercial real estate valuations because the indices there are appraisal based, a potentially good line of modelling in the context of real estate derivatives is described in van Bragt et al. (2015), see also an earlier report van Bragt et al. (2009) or Tunaru (2017). They consider the observed real estate price index as the convex combination of an “efficient market” price or true market price $y(t)$ and the previously observed market price $a(t-1)$

$$a(t) = Ky(t) + (1-K)a(t-1)$$

with K a confidence parameter linking the two. This model is equivalent to an exponentially weighted moving average (EWMA) model that is well-known in financial risk management. To account properly for time value of money the model is adjusted using an expected annual return π

$$a(t) = Ky(t) + (1-K)(1+\pi)a(t-1)$$

van Bragt et al. (2015) assume that the underlying market returns follow a random walk process with drift. For a total return real estate index, they prove that the price of a

forward contract would then be equal to

$$F_t(T) = \frac{1}{df(t, T)} [y(t)(1 - \alpha_{K,T}(t) + a(t)\alpha_{K,T}(t)]$$

where $\alpha_{K,T}(t) = (1 - K)^{T-t}$.

Moreover, van Bragt et al. (2015) derive an approximate formula for the forward *and* a European put option contingent on a real estate index, using techniques developed for pricing Asian options and based on calculate the first moment M_1 and the second moment M_2 of $a(t)$, under the risk-neutral measure. Thus, the forward price formula is

$$F_t(T) = M_1;$$

while the European put option formula for strike X is

$$p(t) = df(t, T) [\Phi(-d_2) - F_t(T)\Phi(-d_1)]$$

with $\sigma = \sqrt{\frac{1}{T-t} \ln\left(\frac{M_2}{M_1}\right)}$, $d_1 = \frac{\ln(F_t(T)/X) + 0.5\sigma^2(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$.

The model developed by van Bragt et al. (2015) can also be adapted to include seasonality effects and there are analytical formulae for pricing swaps on real-estate index as well.

11.5 Forecasting Measures

The root mean squared error (RMSE) is defined as the squared root of the average squared forecasting errors. The mean absolute error (MAE) is defined as the average of the absolute values of forecasting errors. A lower RMSE or MAE indicates a better forecasting method or model. The forecasting measures may be higher because of a couple of really bad forecasts or outliers.

An improved approach for comparing forecasting methods (models) is the Diebold Mariano test Diebold & Mariano (1995). This test is based on a loss function L for the forecasting error e_t and it calculates the loss differential between two methods 1 and 2 as

$$d_{12t} = L(e_{1t}) - L(e_{2t})$$

Under the appropriate technical assumptions the Diebold-Mariano statistic is defined as

$$DM_{12} = \frac{\overline{d_{12}}}{\widehat{\sigma}_{\overline{d_{12}}}}$$

where $\overline{d_{12}} = \frac{1}{T} \sum_{t=1}^{t=T} d_{12t}$ and $DM \rightarrow N(0, 1)$.

The null hypothesis is that the two models produce equal expected forecast loss. The alternative is that one model has a superior (lower) expected forecast loss than the other

one. Usually a quadratic loss function is used, i.e. $L(e_t) = e_t^2$.

11.6 Workshop on NNEG on 28 January 2019

On 28 January 2019, I organised a Workshop on NNEG Valuations at CEQUFIN, Kent Business School, University of Kent.

Participants: Dr Daniel Alai, Dr Jaideep Oberoi, Dr Pradip Tapadar, Dr Vali Asimit, Professor Radu Tunaru, Enoch Quaye.

We had interesting discussions on the current state-of-the-art NNEG valuation and ERM modelling. There was some consensus that the experience learned in dealing with ERMs in other countries may be useful in streamlining modelling efforts on the NNEG valuation for the UK market.

It was widely recognised that this asset class presents characteristics that make any financial modelling challenging but, at the same time, participants were aware (and some of participants were the authors) of many suitable models proposed to value NNEGs, with advantages and disadvantages.

Here are some of the main ideas that came out of that meeting.

1. As the underlying market is incomplete, pricing NNEG based on risk-neutral concepts will always be controversial. It might also be worthwhile to consider the full distribution of the outcomes, based on the real-world projections of the underlying economic variables. I am sure there are plenty of finance/economic/actuarial models suitable for your purpose. Then it will be easier for the reader of your work, to get a good grasp of the “price of NNEG” you propose, within the context of the full underlying distribution. (Pradip Tapadar)
2. I was wondering why you multiply the rental yield by the proportion of properties that are rented out. In other words, why is 5.1776% divided by 5. I just do not see how it is relevant whether other properties are being rented out or not in determining the appropriate rental yield for a certain property. The 80% that are not rented out presumably could be rented out and could provide 5.1776% yield (on average). (Daniel Alai)
3. It would be good to see crossover point type calculations, particularly for cash-flows calculated at portfolio level. (Daniel Alai)
4. It would be really helpful if you could get some data on ERM products from insurers to do an in-house calibration of various methods. Specifically, if the deferment rate is considered important, then a useful estimate of this rate could be obtained from the prices of home reversion agreements that were offered for some time in the market. To estimate this, a method such as the one employed by Cocco and Lopes (2015) might be useful. (Jaideep Oberoi)

5. Basis risk is very important and future research may consider it in an extended methodology. (Jaideep Oberoi)
6. The Federal Housing Administration in the US uses indicative projections of house prices to determine maximum allowable loan to value ratios. (Jaideep Oberoi)
7. It is important to consider unbundled sources of risk to determine their impact on the NNEG, especially given issues with individual risks being modelled separately with some degree of estimation uncertainty. (Jaideep Oberoi)
8. It is better to use a more complex statistical model such as ARMA-EGARCH that fits house prices well than use a computationally simpler model like GBM/Black-Scholes that does not fit well house prices. (Vali Asimit)
9. Fitting an ARMA-EGARCH model and risk-neutralise paths with the conditional Esscher transform is not a very complex task and both are frequently used in actuarial modelling. (Vali Asimit)

11.7 ARMA-GJR Results

We have also redone the analysis using the ARMA-GJR model. Here are the main outputs showing that again a model of this type would be preferable to the GBM model. The graphs in Figures 25 and 26 indicate that the ARMA(4,2)-GJR(1,1) model will outperform the GBM model for two year and five year forecasting horizon.

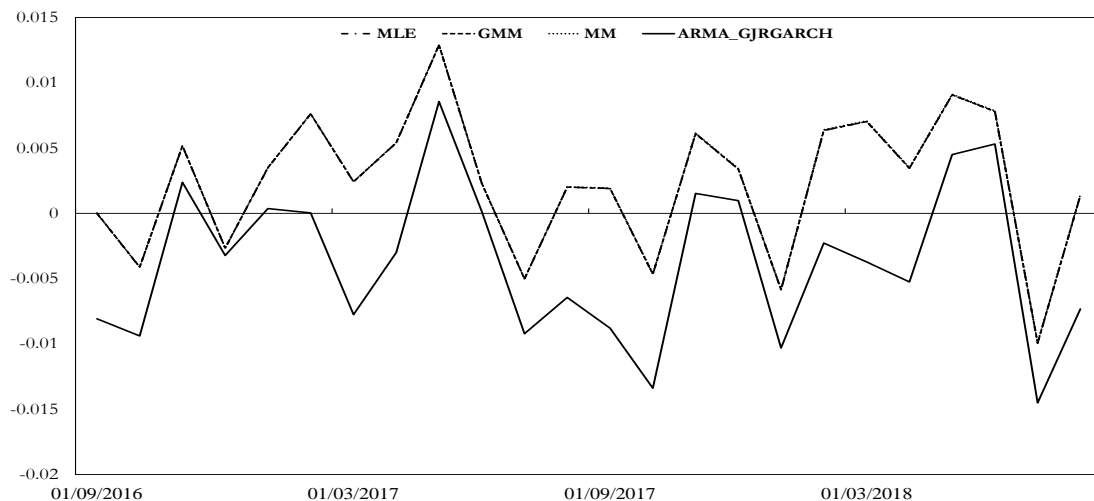


Figure 25: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,2)-GJR(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

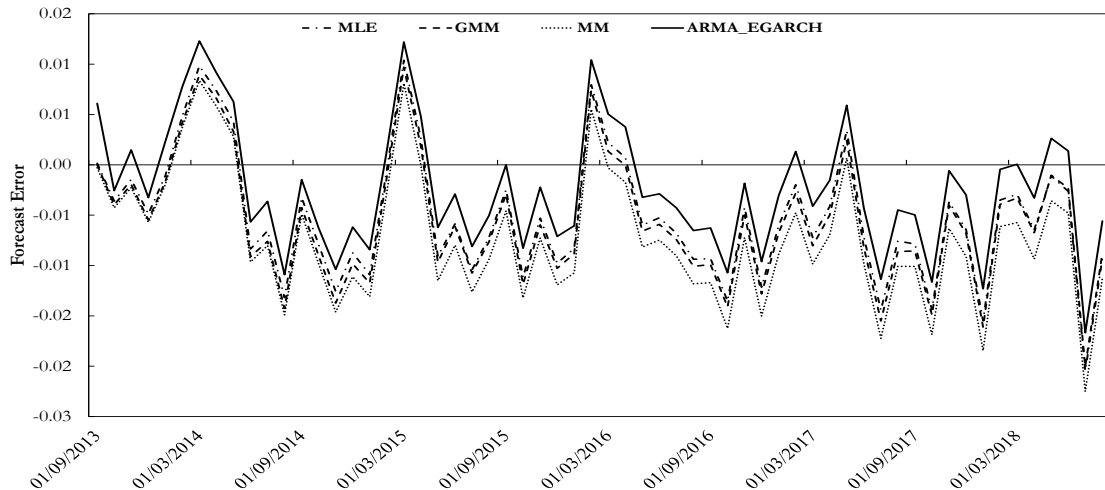


Figure 26: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,2)-GJR(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

Regarding the NNEG valuations, the NNEG values depicted in Figure 27 for the baseline scenario shows a similar performance to the ARMA-EGARCH. For comparison we also report the valuations for the 2014 versus 2018 scenario calculations, in Figure 28.

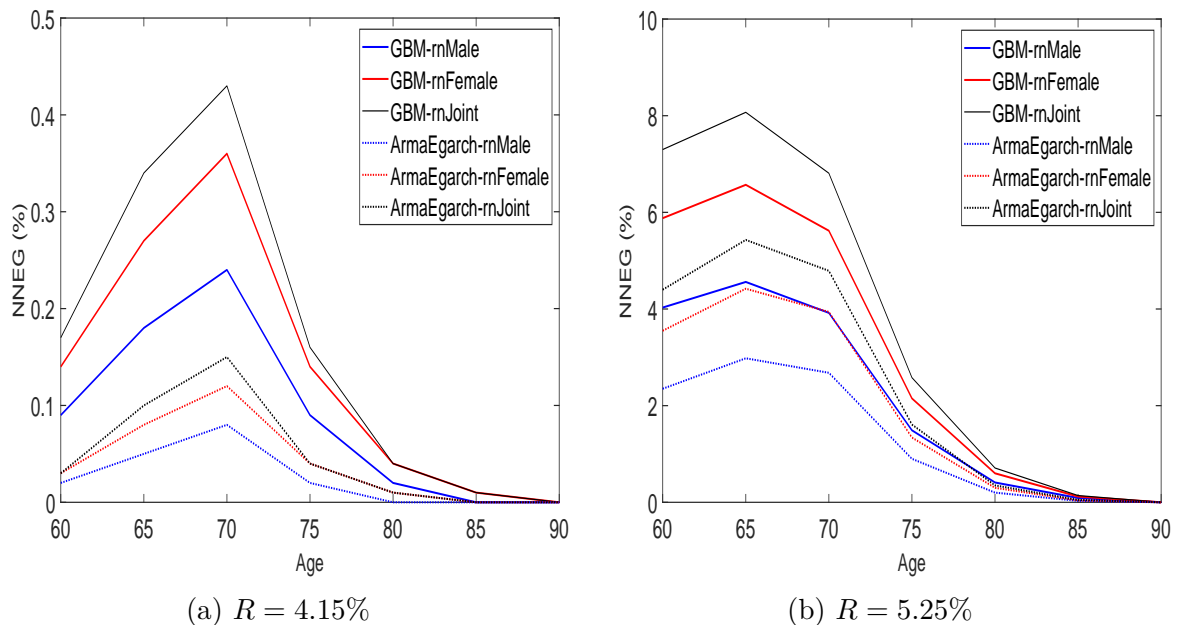


Figure 27: NNEG valuations as percentage of lump sum for GBM-rn and Arma-GJR-rn, under multiple decrement rates for the two baseline scenario with $r = 1.75\%$, $g = 1\%$, $\sigma = 3.90\%$ and standard Flexible LTV vector valuations.

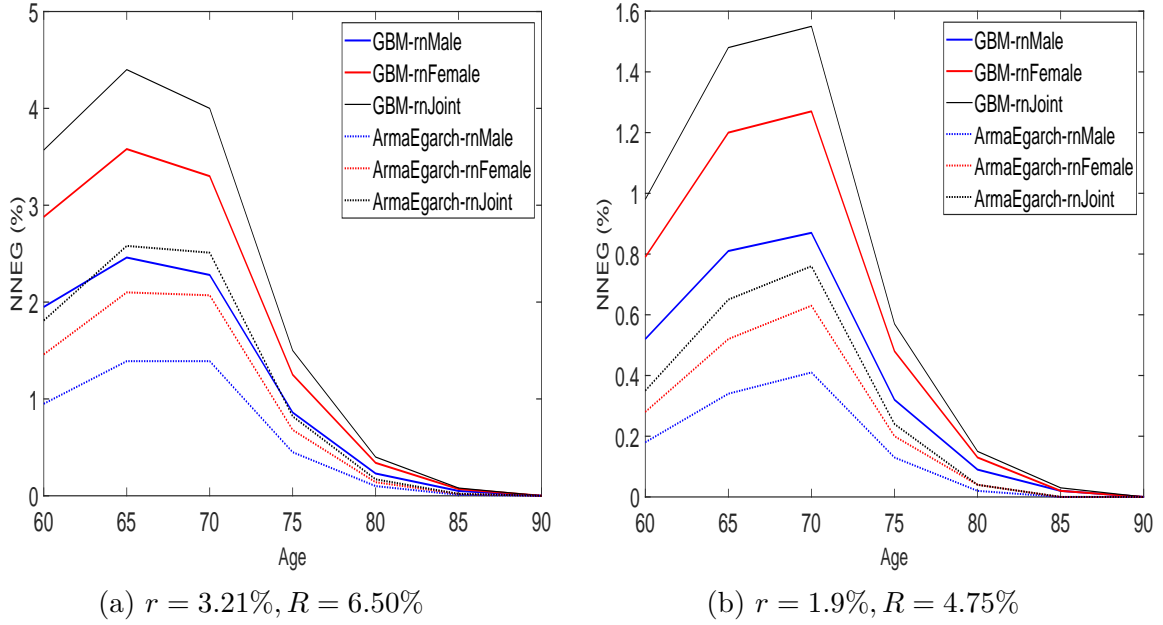
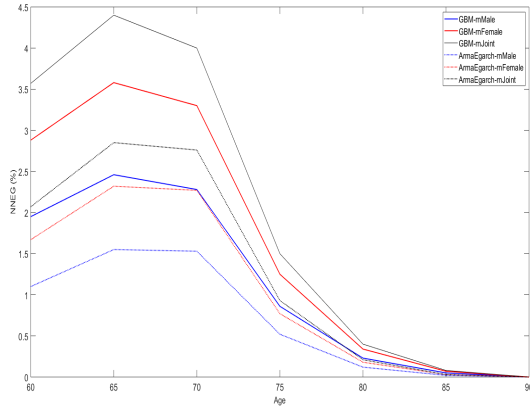


Figure 28: NNEG valuations as percentage of lump sum for GBM-rn and Arma-GJR-rn, under multiple decrement rates for the two baseline scenario with $g = 1\%$, $\sigma = 3.90\%$ and standard Flexible LTV vector valuations.

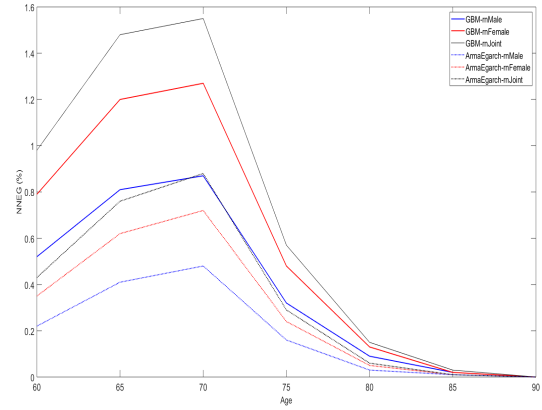
12 Additional Simulation Results

12.1 NNEGs at different points in time

We consider two baseline scenarios, one representative for 2014 when the average risk-free rate taken as proxy with the 20-year swap rate ($r = 3.21\%$) and the average ERC roll-up rate $R = 6.50\%$, and another representative for 2018, with the average 20-year swap rate $r = 1.89\%$ and the average ERC roll-up rate $R = 4.75\%$. The other inputs are taken as identical, although obviously some variations may have existed.



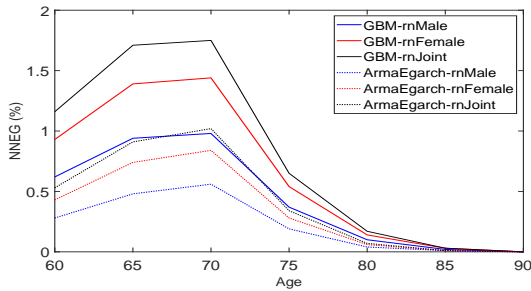
(a) Baseline 2014



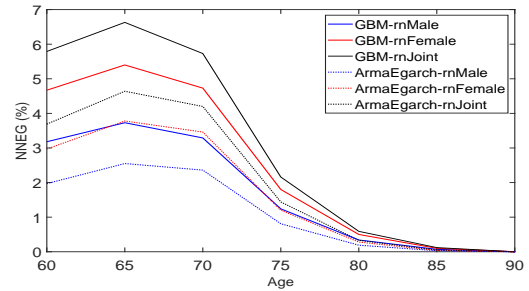
(b) Baseline 2018

Figure 29: NNEG valuations as percentage of lump sum for GBM-rn and Arma-Egarch-rn, under multiple decrement rates for the two baseline scenarios in 2014 and 2018 with $g = 1\%$, $\sigma = 3.90\%$ and standard Flexible LTV vector valuations.

12.2 Legal & General London and South East LTV loading

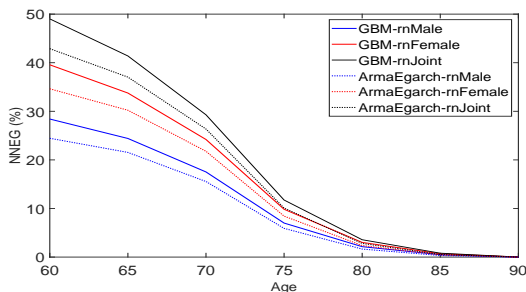


(a) $r = 1.25\%$

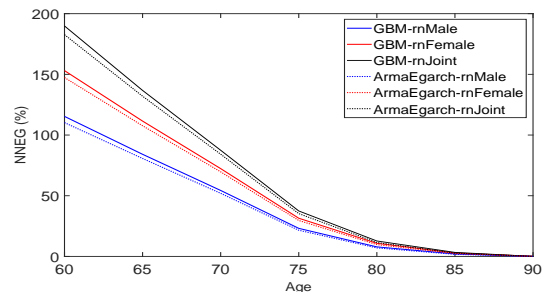


(b) $r = 0.75\%$

Figure 30: Sensitivity Analysis of NNEG valuation w.r.t. r under London and South East LTV loading and $R = 4.13\%$, $g = 1\%$, $\sigma = 3.90\%$.

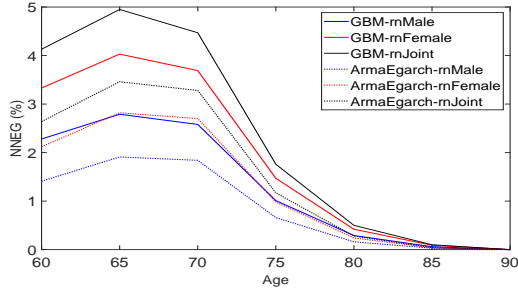


(a) $R = 6.15\%$

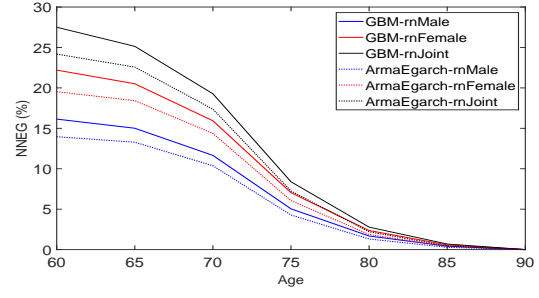


(b) $R = 7.15\%$

Figure 31: Sensitivity Analysis of NNEG valuation w.r.t. r under London and South East LTV loading and $r = 1.75\%$, $g = 1\%$, $\sigma = 3.90\%$.



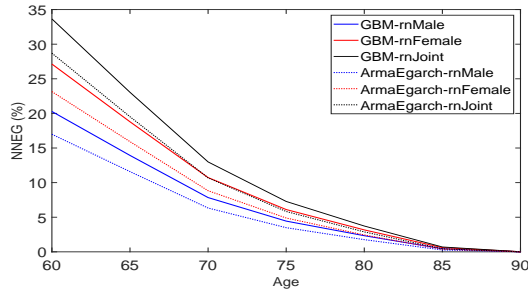
(a) $g = 2\%$



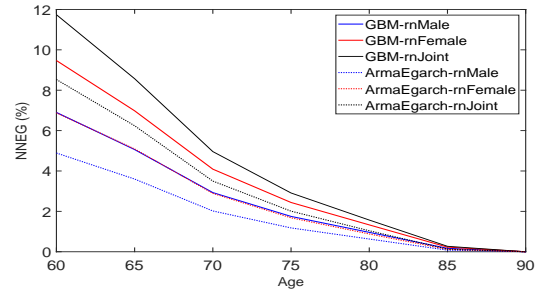
(b) $g = 3\%$

Figure 32: Sensitivity Analysis of NNEG valuation w.r.t. g under London and South East LTV loading and $r = 1.75\%$, $R = 4.13\%$, $\sigma = 3.90\%$.

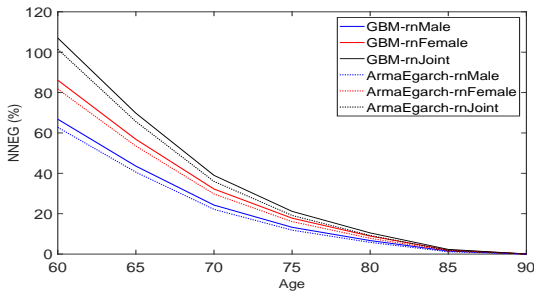
12.3 Flexible Max LTV loading



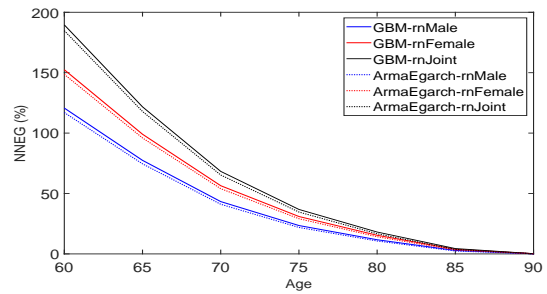
(a) $r = 2\%$



(b) $r = 2.5\%$



(c) $r = 1.25\%$



(d) $r = 0.75\%$

Figure 33: Sensitivity Analysis of NNEG valuation w.r.t. r under Flexible Max LTV loading and $R_{fm} = 4.99\%$, $g = 1\%$, $\sigma = 3.90\%$.

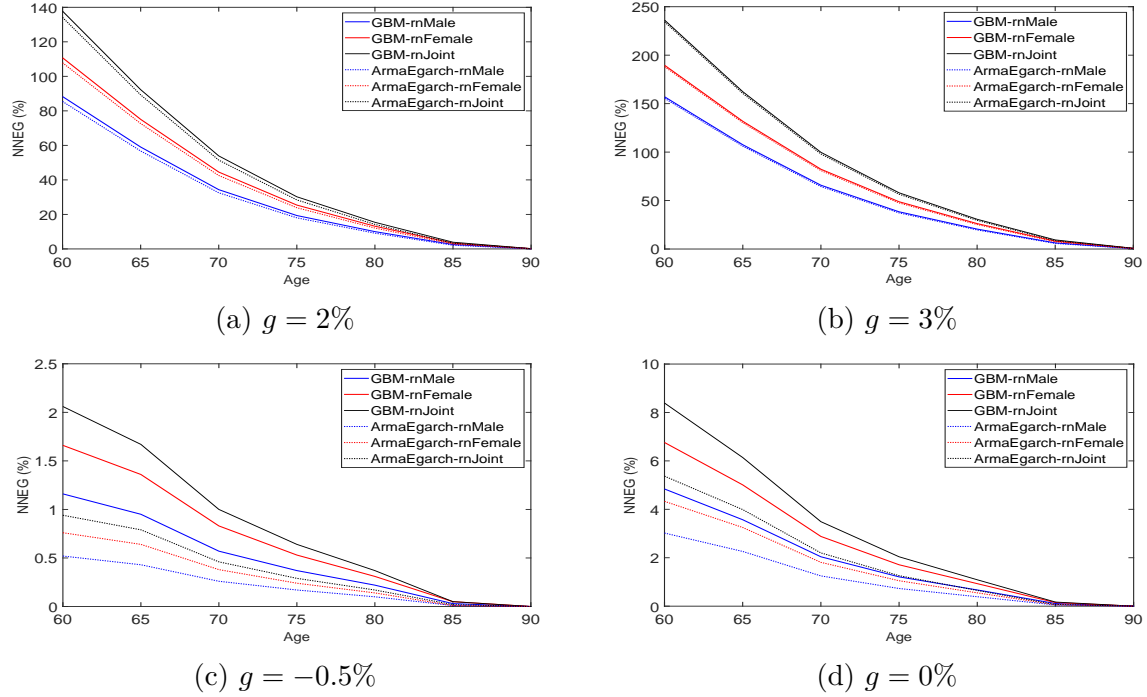


Figure 34: Sensitivity Analysis of NNEG valuation w.r.t. g under Flexible Max LTV loading and $r = 1.75\%$, $R_{fm} = 4.99\%$, $\sigma = 3.90\%$.

12.4 Max ERC LTV loading

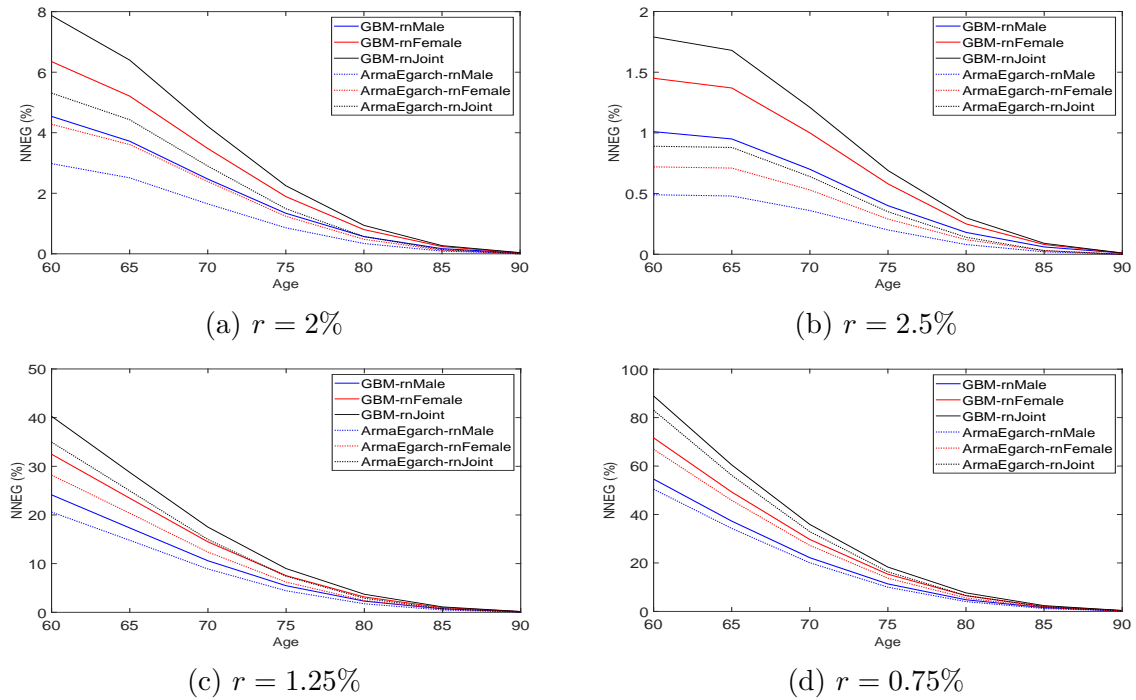


Figure 35: Sensitivity Analysis of NNEG valuation w.r.t. r under Max ERC LTV loading and $R_{max} = 4.56\%$, $g = 1\%$, $\sigma = 3.90\%$.

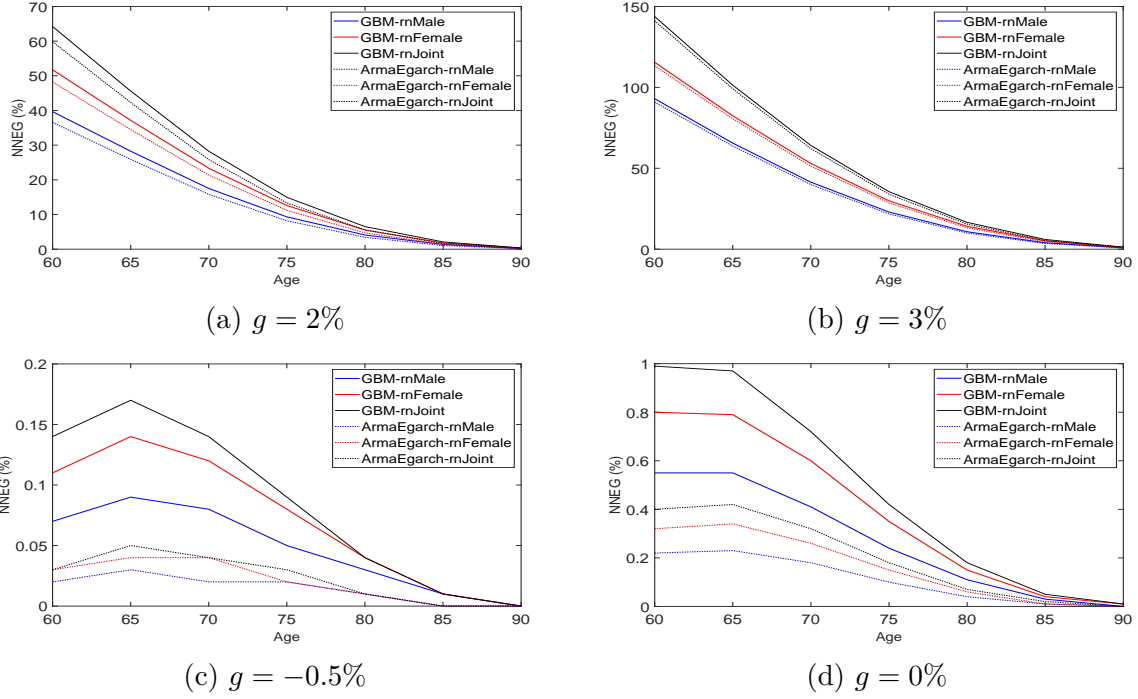


Figure 36: Sensitivity Analysis of NNEG valuation w.r.t. g under Max ERC LTV loading and $r = 1.75\%$, $R_{max} = 4.56\%$, $\sigma = 3.90\%$.

12.5 ERC-Lite

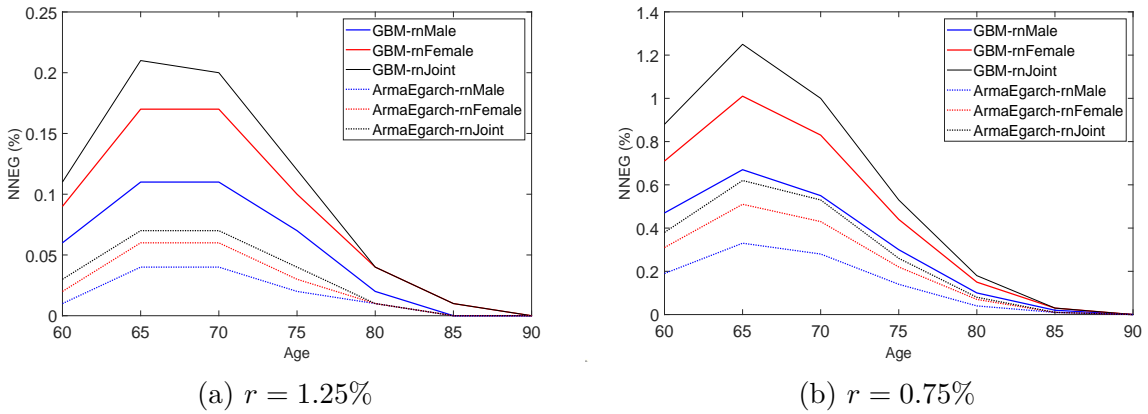
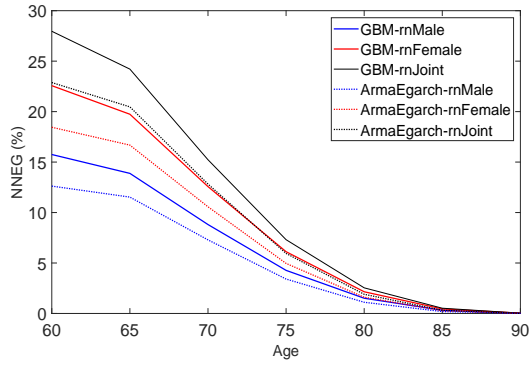
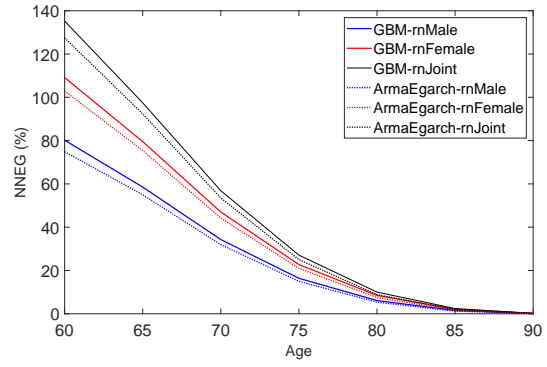


Figure 37: Sensitivity Analysis of NNEG valuation w.r.t. r under ERC-Lite LTV loading and $R_{lite} = 3.85\%$, $g = 1\%$, $\sigma = 3.90\%$.

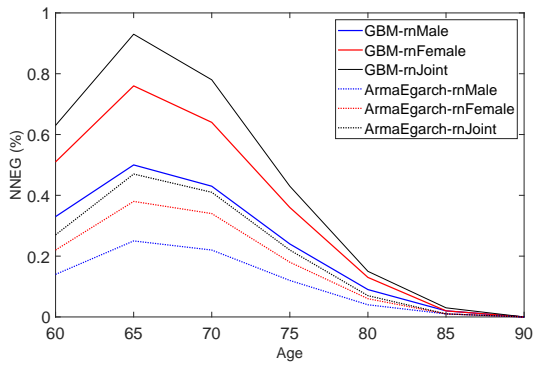


(a) $R = 6.15\%$

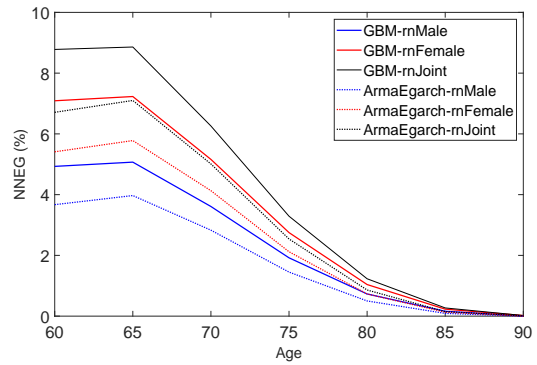


(b) $R = 7.15\%$

Figure 38: Sensitivity Analysis of NNEG valuation w.r.t. R under ERC-Lite LTV loading and $r = 1.75\%$, $g = 1\%$, $\sigma = 3.90\%$.



(a) $g = 2\%$



(b) $g = 3\%$

Figure 39: Sensitivity Analysis of NNEG valuation w.r.t. g under ERC-Lite LTV loading and $r = 1.75\%$, $R_{lite} = 3.85\%$, $\sigma = 3.90\%$.

12.6 Comparison of NNEGs for increasing R

Here we compare the NNEGs under the two approaches, on the log-scale, for increasing roll-up rate R .

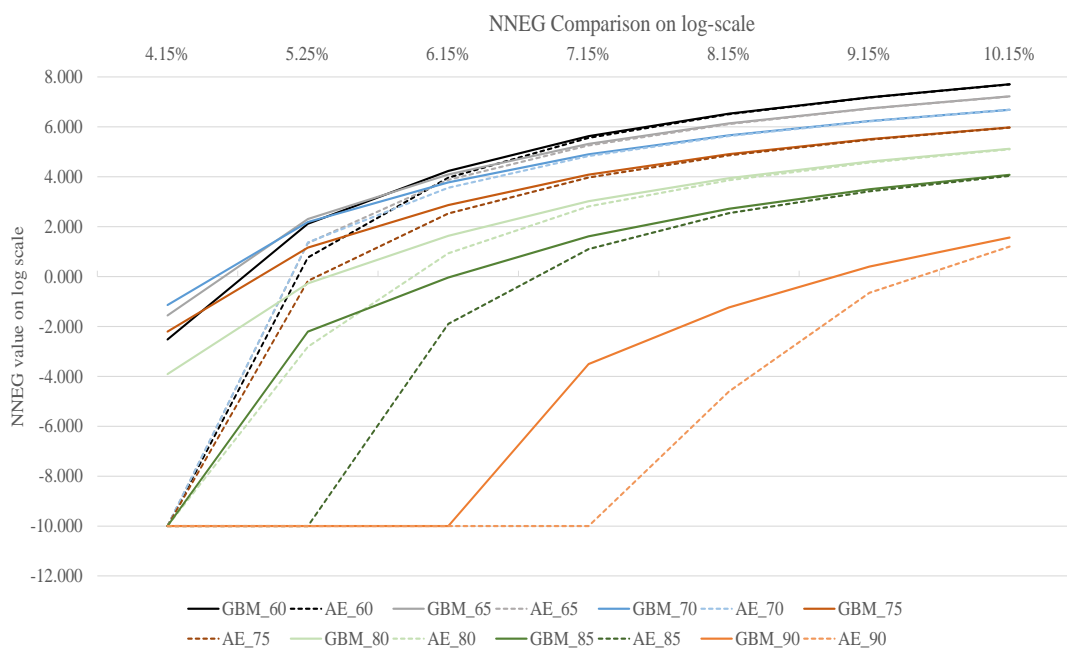


Figure 40: Comparison of NNEGs for increasing R , Flexible LTV and $r = 1.75\%$, $\sigma = 3.90\%$.

13 Cash Flows Analysis

In this section, we follow the cash flows for a hypothetical portfolio of ERMs with male and female borrowers of various ages and various loan characteristics.

Table 21: Descriptive statistics for a hypothetical ERM portfolio.

AgeBand	Prop Value (mil GBP)	Initial Loan (mil GBP)	Weight	Average Maturity
MALE				
<60	16.74	2.30	0.85%	27.69
60-64	204.29	38.80	11.28%	24.59
65-69	440.82	109.20	26.30%	21.01
70-74	447.95	135.87	28.73%	15.78
75-79	264.12	89.63	18.09%	8.02
80-84	141.98	54.52	10.36%	3.00
84+	56.73	23.54	4.39%	3.00
FEMALE				
<60	14.26	1.92	0.75%	28.20
60-64	198.71	37.79	11.31%	25.19
65-69	427.18	105.76	26.26%	22.41
70-74	451.05	136.99	29.83%	18.35
75-79	262.88	89.51	18.58%	12.55
80-84	121.52	46.80	9.15%	4.57
84+	51.77	21.48	4.13%	3.00

13.1 General Set-up

In this section, we will consider the portfolio effects when looking at the cash flows analysis. To this end we considered several portfolios characterised by various features, trying to stay as close as possible with the portfolio assumptions in Hosty et al. (2008). Our main portfolio takes the view that all borrowers live exactly to their expected lifetime. Note that the lifetime expectancy of a 60 year old will differ from the lifetime expectancy of a 70 year old and so on, and it will be also different between males and females.²⁴ The portfolio has 10,000 loan contracts in total, 4927 female and 5073 males. The initial property value is distributed as follows

- 100k - 2500 borrowers on Flexible LTV
- 200k - 2500 borrowers on Flexible LTV
- 310k - 2500 borrowers on Flexible LTV
- 950k - 2500 borrowers on Flexible Max Plus LTV

We are going to follow the following cash flow variables, where i denotes the loan number and t the year ahead.

- N_t = number of loans (males/females) terminating during time t
- $K_t^{(i)}$ = accumulated balance for loan i at time t
- $K_t = \sum_i K_t^{(i)}$ is the portfolio outstanding balance at time t
- $\tau_t^{(i)}$ = indicator variable if termination for loan i arrives in year t (taken as 1) or not (taken as 0)
- $C_t^{(i)} = \min(H_t^{(i)}, K_t^{(i)}) \times \tau_t^{(i)}$ is the cash obtained in year t from loan i
- $C_t = \sum_i C_t^{(i)}$ money in cash account at time t ; this is replenished every time a loan is terminated
- AC_t = portfolio accrued cash at time t ; this is calculated recursively $AC_t = AC_{t-1}(1+r) + C_t$.
- $P_t = \sum_i P_t^{(i)}$ is the total payment expected on the portfolio on year t , $P_t^{(i)} = K_t^{(i)} \times \tau_t^{(i)}$.
- $P_t^{4Y} = P_{t+1} + P_{t+2} + P_{t+3} + P_{t+4}$

²⁴Another portfolio randomises arrival of termination event between the current age of the borrower and 100, so for example for a 65 year old female we draw a random number between 1 and 35. As an extreme portfolio we also consider cash-flows for a portfolio where all borrowers go to 100 years. The results for this latter portfolio are not reported here but they are available from authors upon request.

- $H_t = \sum_i H_t^{(i)}$ = total expected value of house collateral at time t
- $EAR_t^{(i)} = E(\max[K_t^{(i)} - H_t^{(i)}, 0])$ NNEG exposure at risk for loan (i) at time t
- $EAR_t = \sum_i EAR_t^{(i)}$ = total NNEG exposure at risk due to house collateral

Dilapidation costs, together with other termination costs such as sale fees, legal fees, taxation fees, etc. are considered as a haircut rate θ that is applied to the terminal value of the house. We made portfolio calculations with $\theta = 0$ and $\theta = 10\%$.

Following Hosty et al. (2008), we adjust some of the inputs to reflect various socio-economic factors.

13.2 Portfolio Calculations with GBMrn and ARMA-EGARCH

The calculations are based on the assumption that the loans are terminated exactly at the expected future lifetime maturities, for male and female borrowers.

The graphs in Figure 41 show the payments due in year t on the loan in the surviving portfolio. Making calculations under GBM house price modelling the peak for male borrowers will be in year 4 while the peak for female borrowers will be in year 23, followed closely by 19. After roughly 30 years the dynamics in this portfolio does not expect any payments/terminations.

A different picture is portrayed however, for the total payments due over the next four years. Loans for males coming to an end seem to peak after 13-15 years and for females after 18 years.

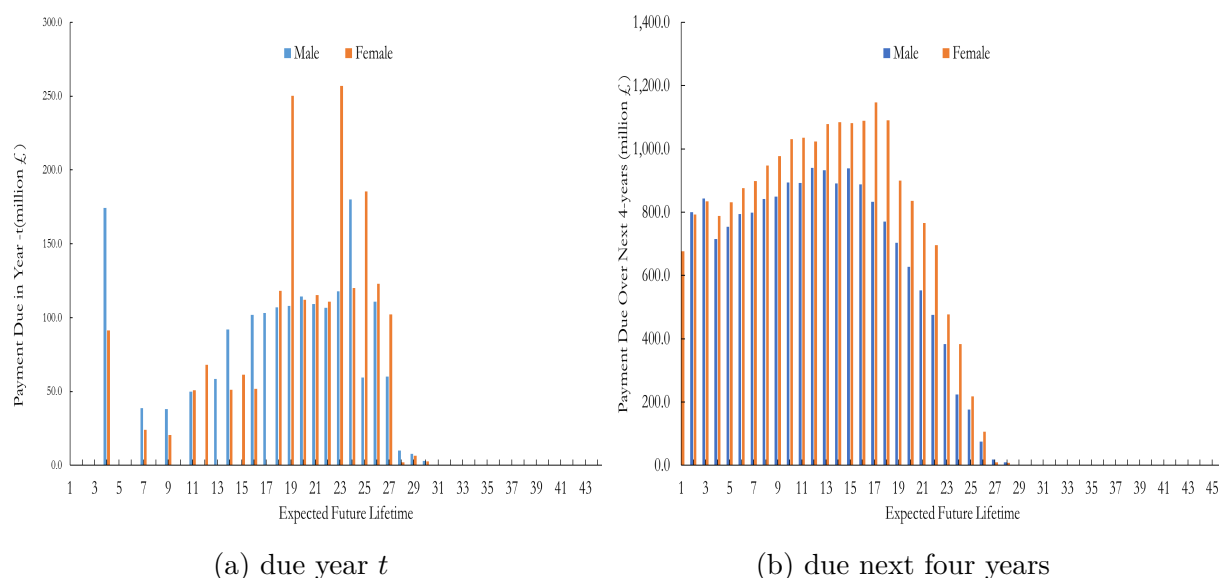
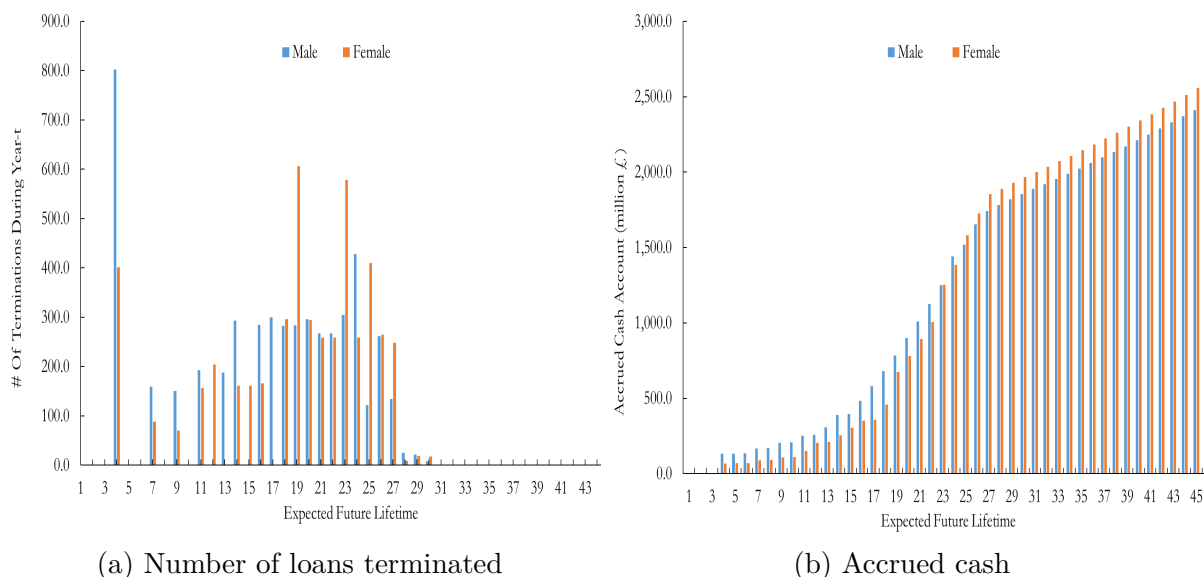


Figure 41: Payments profiles due at various horizons.

In Figure 42, we show the evolution of the number of loans terminated and the cash collected from terminated loans with interest paid on the money account at the risk-free

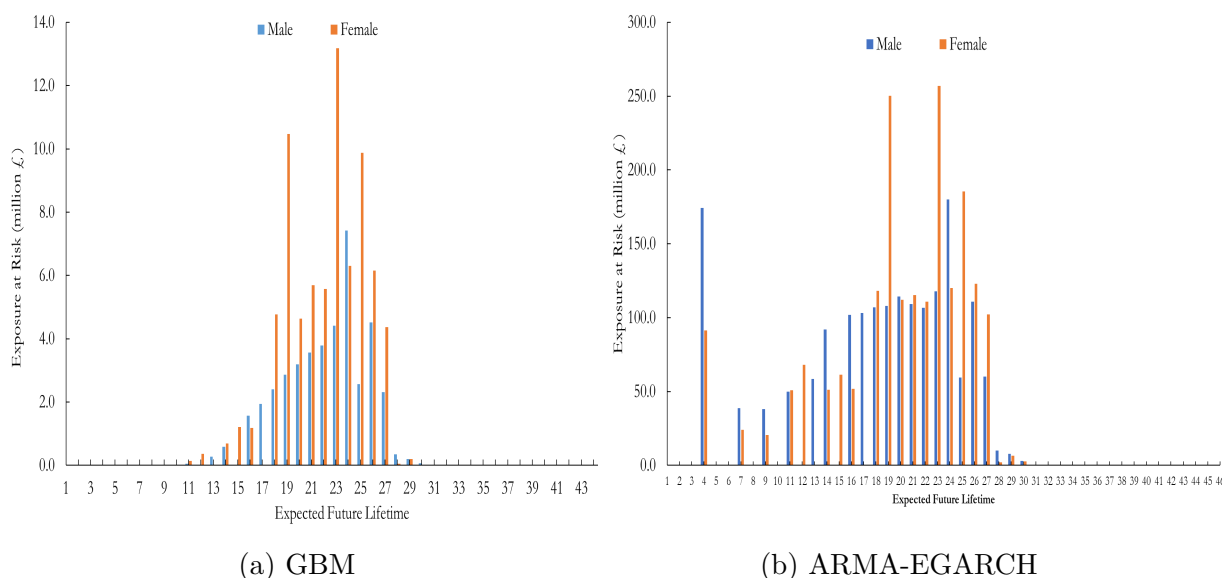
rate. The accrued cash starts slow and it ramps up more abruptly under the expected life approach, between year 10 and 25, with more collections coming from male borrowers.



(a) Number of loans terminated

(b) Accrued cash

Figure 42: Number of loan terminations from portfolio and the accrued cash collected from loans, including interest at risk-free rate, based on expected life for terminations.



(a) GBM

(b) ARMA-EGARCH

Figure 43: EAR portfolio measures, male and female borrowers, using expected life for termination.

The NNEG risk is finely captured in the graphs in Figure 43. This important risk is dominated by female borrowers as expected, since they live longer, and it is high in the region between 18 years and 27 years, if calculations are made based on expected lifetime, under GBM modelling of the house prices. Under ARMA-EGARCH model, the risk manifests itself earlier and it is overall one order of magnitude greater. The reason

for this is that under GBM model, due to inflated variances at the long-term maturities, house prices are very large, offsetting the NNEG risk. On the contrary, the ARMA-EGARCH model keeps house price growth more subdued, implying that NNEG risk may be larger and for a larger part of the portfolio. Hence, under real-world valuation the ARMA-EGARCH model may give higher NNEG values than the GBM real world NNEGs. Recall that the opposite was highlighted for risk-neutral calculations. The explanation is that the risk-neutral drifts are changing the direction of the projected house prices while volatilities stay the same.

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